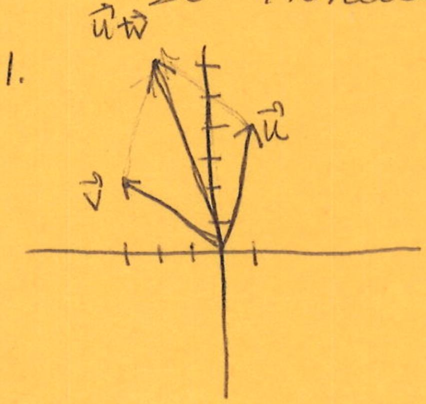
202 Homework #4 key

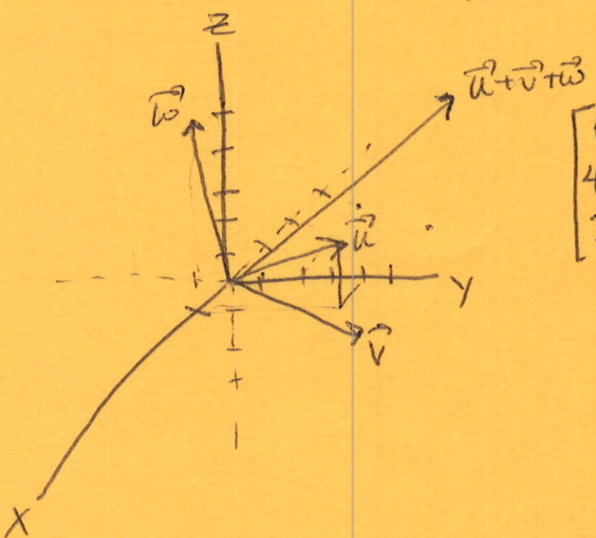
1.



$$\vec{u}+\vec{v} = \begin{bmatrix}1\\4\end{bmatrix} + \begin{bmatrix}-3\\2\end{bmatrix} = \begin{bmatrix}-2\\6\end{bmatrix}$$

if you imagine adding a copy of $\vec{v}$ to the end of $\vec{u}$ (and likewise a copy of $\vec{u}$ to the end of $\vec{v}$) we get a parallelogram and $\vec{u}+\vec{v}$ is the diagonal.

2.



$$\begin{bmatrix}1\\4\\2\end{bmatrix} + \begin{bmatrix}-3\\2\\-3\end{bmatrix} + \begin{bmatrix}0\\-1\\5\end{bmatrix} = \begin{bmatrix}-2\\5\\4\end{bmatrix}$$

3. a. $\vec{a}-\vec{b} = \begin{bmatrix}-1\\2\end{bmatrix} - \begin{bmatrix}3\\5\end{bmatrix} = \begin{bmatrix}-4\\-3\end{bmatrix}$

b. $3\vec{b}+2\vec{a} = 3\begin{bmatrix}3\\5\end{bmatrix} + 2\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}9\\15\end{bmatrix} + \begin{bmatrix}-2\\4\end{bmatrix} = \begin{bmatrix}7\\19\end{bmatrix}$

c. $\vec{c}+2\vec{d}-4\vec{e} = \begin{bmatrix}2\\7\\1\end{bmatrix} + 2\begin{bmatrix}1\\-1\\0\end{bmatrix} - 4\begin{bmatrix}4\\-5\\2\end{bmatrix} = \begin{bmatrix}2\\7\\1\end{bmatrix} + \begin{bmatrix}2\\-2\\0\end{bmatrix} + \begin{bmatrix}-16\\20\\-8\end{bmatrix} = \begin{bmatrix}-12\\25\\-7\end{bmatrix}$

4. a. false $f(t)=0$ for all t or its not the zero vector.

b. false it can be in higher dimensional spaces or not an "arrow".

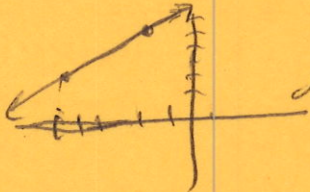
c. false. this is one condition, but not the only one

d. true

e. true

4f. false. R^2 is isomorphic to a subspace of R^3 but $\begin{bmatrix}a\\b\end{bmatrix}$ in R^2 is not in R^3.

g. true when $\vec{u}$ in H.

h. false  line does not go through origin

i. true

j. true

5. rref $\Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{array}\right] \Rightarrow 2\vec{v}_1 + \vec{v}_2 - 2\vec{v}_3 + \vec{v}_4 - \vec{v}_5$

the solution is unique.

6. a. this is not a subspace since b^2 (the second component) is positive and so it will fail scalar multiplication.

$-1\begin{bmatrix}1\\4\end{bmatrix} = \begin{bmatrix}-1\\-4\end{bmatrix}$ but $b^2 \neq -4$ for any real b.

this condition (b^2) is equivalent to saying $y \geq 0$. //

b. This is a subspace. i) if $a = b = 0$, $c = 0$ then $\begin{bmatrix}0\\0\\0\end{bmatrix}$ in V. ii) if $\begin{bmatrix}a\\b\\c\end{bmatrix}$ in V, and $\begin{bmatrix}e\\f\\g\end{bmatrix}$ in V then $\begin{bmatrix}a+e\\b+f\\c+g\end{bmatrix} = \begin{bmatrix}(b+c)+(f+g)\\b+f\\c+g\end{bmatrix} = \begin{bmatrix}(b+f)+(c+g)\\b+f\\c+g\end{bmatrix}$ in V.

$= \begin{bmatrix}(b+c)\\b\\c\end{bmatrix}$ $\quad = \begin{bmatrix}f+g\\f\\g\end{bmatrix}$ since it follows the definition. iii) for

$k\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}ka\\kb\\kc\end{bmatrix} = \begin{bmatrix}k(b+c)\\kb\\kc\end{bmatrix} = \begin{bmatrix}kb+kc\\kb\\kc\end{bmatrix}$ in V. So this is a subspace. //

c. this is not a subspace since $\vec{0}$ not in set. if $a = b = 0$

we get $\begin{bmatrix}0 & 2\\0 & 0\end{bmatrix} \neq \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$. //