

MTH 166 Homework #3 Key

11

1a. $f(x) = -3(x + \frac{1}{2})(x-4)^3$

Zeros $-\frac{1}{2}, 4$

4 has multiplicity 3

both $-\infty, \infty$ go to $-\infty$

b. $f(x) = x^3 + 7x^2 - 4x - 28$

$x^2(x+7) - 4(x+7)$

$(x+7)(x^2-4)$

$(x+7)(x+2)(x-2)$

Zeros $-7, -2, 2$

all multiplicity 1

$-\infty \rightarrow -\infty, \infty \rightarrow \infty$

c. $f(x) = x^4 - x^2$

$x^2(x^2-1)$

$x^2(x-1)(x+1)$

Zeros $0, 1, -1$

0 multiplicity 2

both $-\infty, \infty$ go to ∞

d. $f(x) = 6x^3 - 9x - x^5$

$-x^5 + 6x^3 - 9x$

$-x(x^4 - 6x^2 + 9)$

$-x(x^2-3)^2$

Zeros $0, \pm\sqrt{3}$

$\pm\sqrt{3}$ multiplicity 2 (both)

$-\infty \rightarrow \infty, \infty \rightarrow -\infty$

e. $f(x) = x^2(x-1)^3(x+2)$

Zeros $0, 1, -2$

0 multiplicity 2, 1 multiplicity 3

2a. $a(x+2)(x-1)(x-3) = f(x)$

$f(0) = 12$

$a(2)(-1)(-3) = 12$

$6a = 12$

$a = 2$

$f(x) = 2(x+2)(x-1)(x-3)$

$$2b. a(x+3)x^2(x-2)^3 = f(x)$$

$$f(1) = -6$$

(2)

$$a(1+3)(1)^2(1-2)^3 = -6$$

$$a(4)(1)^2(-1)^3 = -6$$

$$\frac{-4a}{-4} = \frac{-6}{-4} \Rightarrow a = \frac{3}{2}$$

$$f(x) = \frac{3}{2}(x+3)x^2(x-2)^3$$

$$c. f(x) = a(x+2)^3(x+1)(x-2-3i)(x-2+3i)$$

$$f(0) = 24$$

$$x^2 - 2x + 3xi - 2x + 4 - 6i - 3xi + 6i + 9$$

$$a(x+2)^3(x+1)(x^2 - 4x + 13)$$

$$a \frac{(2)^3}{8} (1)(13) = \frac{24}{8} \quad 13a = 3$$

$$a = \frac{3}{13}$$

$$f(x) = \frac{3}{13}(x+2)^3(x+1)(x^2 - 4x + 13)$$

$$d. f(x) = a(x+1)^2(x-1)(x-1-i)(x-1+i)(x-2-i)(x-2+i)$$

$$a(x+1)^2(x-1)(x^2 - 2x + 2)(x^2 - 4x + 5)$$

$$f(0) = 60$$

$$a(1)^2(-1)(2)(5) = 60$$

$$-10a = 60$$

$$a = -6$$

$$f(x) = -6(x+1)^2(x-1)(x^2 - 2x + 2)(x^2 - 4x + 5)$$

$$3a. \begin{array}{r} x+3 \\ x+5 \overline{) x^2 + 8x + 15} \\ \underline{-x^2 + 5x} \\ 3x + 15 \\ \underline{-3x + 15} \\ 0 \end{array}$$

$$\underline{x+3}$$

long