

MTH 291 Skills Problems #3 Key

(1)

1. it is linear since  $\frac{d^3y}{dt^3}$ ,  $\frac{dy}{dt}$ ,  $y$  are not raised to powers, multiply each other, etc. 3rd order. Ordinary.

2. a.  $y' - 2y = 3e^t$

$\mu = e^{\int 2 dt} = e^{-2t}$

$e^{-2t} y' - 2e^{-2t} y = 3e^t e^{-2t} = 3e^{-t}$

$\int (ye^{-2t})' = \int 3e^{-t} dt \rightarrow ye^{-2t} = -3e^{-t} + C$

$y = -3e^{-t} + Ce^{-2t}$

b.  $ty' - y = -t^2 e^{-t}$

$\rightarrow y' - \frac{1}{t}y = -te^{-t}$

$\frac{1}{t}y' - \frac{1}{t^2}y = e^{-t}$

$\mu = e^{\int -\frac{1}{t} dt} = e^{-\ln t} = \frac{1}{t}$

$\int (y \cdot \frac{1}{t})' = \int e^{-t} dt \rightarrow y \cdot \frac{1}{t} = -e^{-t} + C$

$y = -te^{-t} + Ct$

c.  $t^3 y' + 4t^2 y = e^{-t}$

$y(-1) = 0$

$y' + \frac{4}{t}y = \frac{e^{-t}}{t^3}$

$\mu = e^{\int \frac{4}{t} dt} = e^{4 \ln t} = t^4$

$t^4 y' + 4t^3 y = te^{-t}$

$\int (t^4 y)' = \int te^{-t} dt$

$u = t$   
 $\frac{du}{dt} = 1$   
 $v = -e^{-t}$

$t^4 y = -te^{-t} + \int e^{-t} dt$   
 $= -te^{-t} - e^{-t} + C$

$0 = \frac{+e}{+1} - \frac{e}{1} + \frac{C}{1} \rightarrow C = 0$

$y = -\frac{e^{-t}}{t^3} - \frac{e^{-t}}{t^4} + \frac{C}{t^4}$

$y = -\frac{e^{-t}}{t^3} - \frac{e^{-t}}{t^4}$