MAT 100, Sequences Worksheet Name ______________________________

Sequences can be defined in a couple different ways. They can be defined with an explicit formula, or they can be defined by a recursive formula and (an) initial value(s). Both these methods define a sequence precisely for any term in the sequence so that it can be distinguished from any other sequence. Sometimes we given sequences by setting up a pattern, such as $\{A_0, A_1, A_2, A_3, \ldots\}$ or $\{1, 2, 3, 5, 7, 11, \ldots\}$. This method is always a bit ambiguous, because we can always come up with some complicated formulas that have the same set of initial set of values, but eventually differ from each other.

Sequences that use formulas to define the set of values, uses a formula with subscripts. An explicit formula might look like: $A_N = \frac{3+N}{2N-1}$; or a recursive on might look like $P_N = 2P_{N-1}$. The subscript tells you which term in the sequence you are solving for. In the first case, if you want A_{10}, then you need to plug $N = 10$ in the explicit formula to obtain the value. If we start counting at $N = 1$, then this is the tenth value in the sequence. In the second case, this says that if $N = 10$, then $P_{10} = 2P_9$. To find P_{10}, we'd need to first find P_9. This kind of sequences formula is called recursive because it depends on knowing all the values in the sequence back to the original value.

One other important thing of note: Sequences can differ based on where they start counting. If we start at A_0 or A_1 we use slightly different methods for turning a set of numbers into a formula, so it's important to specify where we are starting. This is sometime done like this: $\{A_N\}_{N=0}$ which says our sequence starts at $N = 0$.

Let's try working with the formulas first.

1. For each sequence formula below, find the next 5 terms of the sequence. Start at $N = 0$ unless otherwise specified.
   a. $A_N = 2^N - 3N$ $A_0 = 1,\ A_1 = -1,\ A_2 = -2,\ A_3 = -1,\ A_4 = 4,\ A_5 = 17$

   b. $A_N = \frac{(N+2)!}{N+1}$ $A_0 = 2,\ A_1 = 3,\ A_2 = 8,\ A_3 = 30,\ A_4 = 144,\ A_5 = 840$

   c. $A_N = A_{N-1} + 2A_{N-2}, A_1 = 1, A_2 = 1$ $A_3 = 1+2(1) = 3,\ A_4 = 3+2(1) = 5,$
   $A_5 = 5+2(3) = 11,\ A_6 = 11+2(5) = 21$

   d. $A_N = -3A_{N-1} + 5$ $A_1 = 1,\ A_2 = -3(1)+5 = 2,\ A_3 = -3(2)+5 = -1$
   $A_4 = -3(-1)+5 = 8,\ A_5 = -3(8)+5 = -19,\ A_6 = -3(-19)+5 = 62$

2. Let's try writing our own formulas. First, verify that these are arithmetic sequences by finding the common difference. The recursive formula is $A_N = A_{N-1} + D$. D can be either positive or negative, so watch the sign.
   a. $3, 7, 11, 15, 19, \ldots$

   $A_N = A_{N-1} + 4,\ A_1 = 3$

   b. $19, \frac{73}{4}, \frac{35}{2}, \frac{67}{4}, 16, \frac{61}{4}, \frac{29}{2}, \ldots$

   $A_N = A_{N-1} - 3/4,\ A_1 = 19$

3. Then use the explicit formula $A_N = DN + A_0$ to write a formula for the sequence for the same sequences. [Note: if we start counting at A_1, then the formula becomes $A_n = D(n-1) + A_1$.]

$A_n = 3 + 4(N-1) = 3 + 4N - 4 = -1 + 4N$

$A_n = 19 - 3/4\,(N-1) = 19 - 3/4\,N + 3/4 = \frac{79}{4} - 3/4\,N$

4. We can do something similar for a geometric sequence. Verify that each of the sequences below is geometric by finding the common ratio, then write a recursive formula for the sequence as $A_N = RA_{N-1}$.

a. $3, 6, 12, 24, 48, \ldots$

$A_N = 2A_{N-1}$, $A_1 = 1$

b. $\frac{15}{2}, 5, \frac{10}{3}, \frac{20}{9}, \ldots$

$A_N = \frac{2}{3} A_{N-1}$, $A_1 = \frac{15}{2}$

5. Now we want to write an explicit formula for each sequence using the formula $A_N = A_0 R^N$.

$A_N = 3 \cdot 2^n$

$A_N = \frac{15}{2}\left(\frac{2}{3}\right)^N$

6. Suppose we want to add up all the terms in the sequence in 2a and (separately) in 4a up to A_{12}. To do this for an arithmetic sequence, we use the formula $S_N = \frac{(A_0 + A_{N-1})N}{2}$. To do this for a geometric sequence, we use the formula $S_N = \frac{A_0(1-R^N)}{1-R}$. Find the sum in both cases.

$\frac{(3+47)13}{2} = 325$

$3\left(\frac{1-(2)^{13}}{1-2}\right) = 24,573$