MAT 142, Exam #1, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Complete all parts of each question. Questions that provide only answers and no work will not receive full credit. If you use your calculator (only when problems don't instruct you to do the problem by hand), showing calculator steps will count as "work".

1. Simplify. $\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{x^2-4}}$ (5 points)

$(x-2)(x+2)$ $(x+2)(x-2)$

$$\frac{3(x+2)-4(x-2)}{7} = \frac{3x+6-4x+8}{7} = \boxed{\frac{-x+14}{7}}$$

2. Solve.

a. $\left[5+\frac{x-2}{3}=\frac{x+3}{8}\right]24$ (4 points)

$120+8(x-2)=3(x+3)$

$120+8x-16=3x+9$

$104+8x=3x+9$

$-104 \quad -104$

$8x=3x-95$

$-3x \quad -3x$

$\frac{5x}{5}=\frac{-95}{5}$

$\boxed{x=-19}$

b. $\frac{4}{x^2+3x-10}-\frac{1}{x^2+x-6}=\frac{3}{x^2-x-12}$ (5 points)

$(x+5)(x-2) \quad (x+3)(x-2) \quad (x-4)(x+3)$

$LCD=(x+5)(x-2)(x-4)(x+3)$

$4(x-4)(x+3)-1(x+5)(x-4)=3(x+5)(x-2)$

$4(x^2-x-12)-(x^2+x-20)=3(x^2+3x-10)$

$4x^2-4x-48-x^2-x+20=3x^2+9x-30$

$3x^2-5x-28=3x^2+9x-30$

$-9x+28 \quad -9x+28$

$\frac{-14x}{-14}=\frac{-2}{-14}$

$\boxed{x=\frac{1}{7}}$

does not make denominator 0 so, it's okay!

3. Solve. Find all real or complex solutions.

a. $x^2 + 4x + 1 = 0$ (4 points)

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)}}{2} = \frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm 2\sqrt{3}}{2} = \boxed{-2 \pm \sqrt{3}}$$

b. $2x^2 - 7x + 3 = 0$ (4 points)

$$(2x-1)(x-3) = 0$$

$$\boxed{x = 1/2,\ x = 3}$$

c. $(2x - 5)(x + 1) = 2$ (5 points)

$$2x^2 + 2x - 5x - 5 = 2$$
$$-2 \quad\quad -2$$
$$2x^2 - 3x - 7 = 0$$

$$x = \frac{3 \pm \sqrt{9 + 56}}{4} = \boxed{\frac{3 \pm \sqrt{65}}{4}}$$

d. $\sqrt{(2x + 8)^2} = \sqrt{27}$ (4 points)

$$2x + 8 = \pm\sqrt{27} = \pm 3\sqrt{3}$$
$$-8 \quad\quad\quad -8$$

$$\frac{2x}{2} = \frac{\pm 3\sqrt{3} - 8}{2}$$

$$\boxed{x = -4 \pm \frac{3}{2}\sqrt{3}}$$

4. Solve.

a. $\left(\sqrt{x + 10}\right)^2 = (x - 2)^2$ (6 points)

$$x + 10 = x^2 - 4x + 4$$
$$-x \quad -10 \quad\quad -x \quad -10$$

$$0 = x^2 - 5x - 6$$
$$(x-6)(x+1) = 0$$
$$x = 6,\ x = -1$$

Check!

$\sqrt{6+10} = \sqrt{16} = 4$? $6 - 2 = 4$ ✓

$\sqrt{-1+10} = \sqrt{9} = 3$? $-1 - 2 = -3$ ✗ no

$$\boxed{x = 6}$$

b. $6x^{5/2} - 12 = 0$ (4 points)

$\frac{6x^{5/2}}{6} = \frac{12}{6}$

$x^{5/2} = 2$

$x = 2^{2/5} = \sqrt[5]{4}$

c. $\left(y - \frac{8}{y}\right)^2 + 5\left(y - \frac{8}{y}\right) - 14 = 0$ (6 points)

$u = y - \frac{8}{y}$

$u^2 + 5u - 14 = 0$

$(u+7)(u-2) = 0$

$u = -7, u = 2$

$\left(y - \frac{8}{y} = -7\right)y$

$y^2 - 8 = -7y \Rightarrow y^2 + 7y - 8 = 0$

$(y+8)(y-1) = 0$ $\boxed{y = -8, y = 1}$

$\left(y - \frac{8}{y} = 2\right)y$

$y^2 - 8 = 2y \Rightarrow y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$ $\boxed{y = 4, y = -2}$

d. $2\left|4 - \frac{5}{2}x\right| + 6 = 18$ (4 points)

$-6 \quad -6$

$\frac{2\left|4 - \frac{5}{2}x\right|}{2} = \frac{12}{2}$

$\left|4 - \frac{5}{2}x\right| = 6$

$4 - \frac{5}{2}x = 6$ $\quad$ $4 - \frac{5}{2}x = -6$

$-4 \quad -4$ $\quad$ $-4 \quad -4$

$\left(-\frac{2}{5}\right)-\frac{5}{2}x = 2\left(-\frac{2}{5}\right)$ $\quad$ $\left(-\frac{2}{5}\right)-\frac{5}{2}x = -10\left(-\frac{2}{5}\right)$

$\boxed{x = \frac{-4}{5}}$ $\quad$ $\boxed{x = 4}$

5. Factor completely. (3 points each)

a. $2x^4 - 162$

$2(x^4 - 81) = 2(x^2 - 9)(x^2 + 9) = 2(x-3)(x+3)(x^2+9)$

b. $x^3 - 125$

$(x-5)(x^2 + 5x + 25)$

c. $(x+5)^{-3/2} - (x+5)^{-9/2}$

$\frac{1}{(x+5)^{3/2}} - \frac{1}{(x+5)^{9/2}} = \frac{(x+5)^3}{(x+5)^{9/2}} - \frac{1}{(x+5)^{9/2}} =$

$\frac{x^3 + 15x^2 + 75x + 125 - 1}{(x+5)^{9/2}} = (x+5)^{-9/2}\left[x^3 + 15x^2 + 75 + 124\right]$

d. $x^4 + 81$

Prime
Sum of squares

6. Find the indicated sum. (4 points each)

a. $\sum_{i=1}^{6} 7i$

$7(1)+7(2)+7(3)+7(4)+7(5)+7(6) =$
$7[1+2+3+4+5+6] = 7(21) = \boxed{147}$

b. $\sum_{i=1}^{5} \frac{(i+2)!}{i!}$

$\frac{3!}{1!} + \frac{4!}{2!} + \frac{5!}{3!} + \frac{6!}{4!} + \frac{7!}{5!} =$

$\frac{6}{1} + \frac{24}{2} + \frac{120}{6} + \frac{720}{24} + \frac{5040}{120} = 6+12+20+30+42$

$= \boxed{110}$

7. Write a formula in summation notation for $\frac{1}{3}+\frac{2}{4}+\frac{3}{5}+\cdots+\frac{16}{18}$. (3 points)

$\sum_{i=1}^{16} \frac{n}{n+2}$

8. Write a formula for the infinite series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots$, then find the sum. (5 points)

$\sum_{i=0}^{\infty} \left(-\frac{1}{2}\right)^i = \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \boxed{\frac{2}{3}}$