

# 142 Homework #4 Key

(1)

1.a.  $f(x) = 5x - 9 \Rightarrow y = 5x - 9 \Rightarrow x = \frac{y+9}{5} \Rightarrow \frac{x+9}{5} = y = f^{-1}(x)$

one-to-one

$5\left(\frac{x+9}{5}\right) - 9 = x + 9 - 9 = x$   $D_{f^{-1}}$ : all reals

b.  $f(x) = x^3 + 2 \Rightarrow y = x^3 + 2 \Rightarrow x = y^3 + 2 \Rightarrow x - 2 = y^3 \Rightarrow y = \sqrt[3]{x-2} = f^{-1}(x)$

one-to-one

$(\sqrt[3]{x-2})^3 + 2 = x - 2 + 2 = x$   $D_{f^{-1}}$ : all reals

c.  $f(x) = \frac{2x+1}{x-3} \Rightarrow y = \frac{2x+1}{x-3} \Rightarrow x = \frac{2y+1}{y-3} \Rightarrow xy - 3x = 2y+1 \Rightarrow$

one-to-one

$xy - 2y = 3x+1 \Rightarrow y(x-2) = 3x+1 \Rightarrow y = \frac{3x+1}{x-2}$

domain  $f^{-1}$ : all reals  $\neq 2$

$\frac{2\left(\frac{3x+1}{x-2}\right) + 1}{\left(\frac{3x+1}{x-2}\right) - 3} \cdot \frac{x-2}{x-2} = \frac{6x+2+x-2}{3x+1-3x+6} = \frac{7x}{7} = x$

d.  $f(x) = \sqrt[3]{x} + 1 \Rightarrow y = \sqrt[3]{x} + 1 \Rightarrow x = \sqrt[3]{y} + 1 \Rightarrow x - 1 = \sqrt[3]{y} \Rightarrow (x-1)^3 = y$

one-to-one

domain  $f^{-1}$ : all reals  $y = \sqrt[3]{(x-1)^3} + 1 = x - 1 + 1 = x$

e.  $f(x) = e^{2x+1} \Rightarrow y = e^{2x+1} \Rightarrow x = \frac{1}{2} \ln y \Rightarrow \ln x = 2y+1 \Rightarrow (\ln x) - 1 = 2y$

one-to-one

$y = \frac{1}{2} [\ln x - 1]$  domain  $f^{-1}$ :  $(0, \infty)$

$x = e^{\frac{1}{2}(\ln x - 1) + 1} = e^{\ln x - 1 + 1} = e^{\ln x} = x$

f. one-to-one

domain  $f^{-1} = [0, 1]$

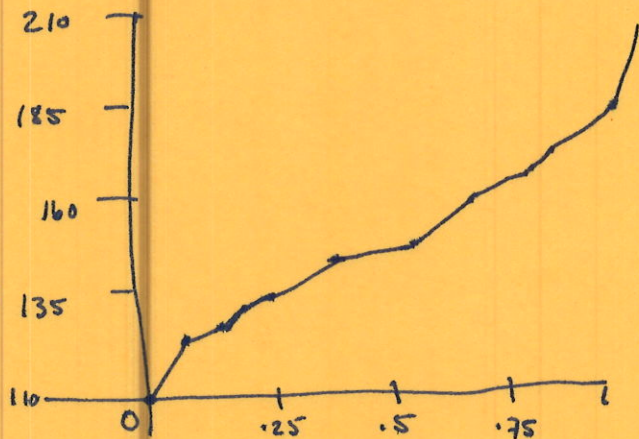
g.  $f(x) = \frac{3}{x-4} \Rightarrow y = \frac{3}{x-4} \Rightarrow x = \frac{3}{y-4}$

one-to-one

$\Rightarrow xy - 4x = 3 \Rightarrow \frac{xy}{x} = \frac{3+4x}{x} \Rightarrow f^{-1} = \frac{3+4x}{x}$

$D_{f^{-1}}$ :  $x \neq 0$

$\frac{3}{\left(\frac{3+4x}{x}\right) - 4} \cdot \frac{x}{x} = \frac{3x}{3+4x-4x} = \frac{3x}{3} = x$



$$1. h. f(x) = \sqrt{x} \Rightarrow y = \sqrt{x} \Rightarrow x^2 = y \quad Df' = x > 0$$

$$\Rightarrow x = \sqrt{y}$$

one-to-one

$$y = (\sqrt{x})^2 = x$$

$$i. f(x) = x^2 - 4 \Rightarrow y = x^2 - 4 \Rightarrow x = y^2 - 4 \quad \text{restrict domain to } x > 0$$

$$x + 4 = y^2 \Rightarrow \sqrt{x+4} = y \quad \text{Domain } f^{-1}: x > -4$$

$$(\sqrt{x+4})^2 - 4 = x + 4 - 4 = x$$

$$j. f(x) = x^2 + x + 2 \Rightarrow y = x^2 + x + \frac{1}{4} - \frac{3}{4} = (x + \frac{1}{2})^2 - \frac{3}{4} \quad \text{restrict domain to } x > -\frac{1}{2}$$

$$x = (y + \frac{1}{2})^2 - \frac{3}{4} \Rightarrow x - \frac{3}{4} = (y + \frac{1}{2})^2 \Rightarrow$$

$$\sqrt{x - \frac{3}{4}} = y + \frac{1}{2} \Rightarrow y = \sqrt{x - \frac{3}{4}} - \frac{1}{2} \quad \text{domain } x > \frac{3}{4}$$

$$(\sqrt{x - \frac{3}{4}} - \frac{1}{2})^2 + \sqrt{x - \frac{3}{4}} + \frac{1}{2} + 2 = x - \frac{3}{4} - \sqrt{x - \frac{3}{4}} + \frac{1}{4} + \sqrt{x - \frac{3}{4}} - \frac{1}{2} + 2$$

$$= x - \frac{3}{4} - \frac{1}{4} + 2 = x - 2 + 2 = x$$

$$k. f(x) = 3 \ln(x+1) \Rightarrow y = 3 \ln(x+1) \Rightarrow x = 3 \ln(y+1) \Rightarrow \frac{x}{3} = \ln(y+1) \Rightarrow$$

$$e^{x/3} = y+1 \Rightarrow e^{x/3} - 1 = y \quad D: \text{all reals}$$

$$3 \ln(e^{x/3} - 1 + 1) = 3 \ln(e^{x/3}) = 3(x/3) = x$$

$$2a. \frac{3(x+h)+7 - (3x+7)}{h} = \frac{3x+3h+7-3x-7}{h} = \frac{3h}{h} = h$$

$$b. \frac{(x+h)^2 - 4(x+h) + 3 - (x^2 - 4x + 3)}{h} = \frac{x^2 + 2xh + h^2 - 4x - 4h + 3 - x^2 + 4x - 3}{h} \\ = \frac{h(2x+h-4)}{h} = 2x+h-4$$

$$c. \frac{b-b}{h} = 0$$

$$d. \frac{(\sqrt{x+h-1} - \sqrt{x-1})(\sqrt{x+h-1} + \sqrt{x-1})}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{x+h-1 - (x-1)}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{x+h-1-x+1}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{h}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{1}{\sqrt{x+h-1} + \sqrt{x-1}}$$