

# 142 Homework #4 Key

①

1.a.  $f(x) = 5x - 9 \Rightarrow y = 5x - 9 \Rightarrow x = \frac{y+9}{5} \Rightarrow \frac{x+9}{5} = y = f^{-1}(x)$

one-to-one

$5\left(\frac{x+9}{5}\right) - 9 = x + 9 - 9 = x$   $D_{f^{-1}}$ : all reals

b.  $f(x) = x^3 + 2 \Rightarrow y = x^3 + 2 \Rightarrow x = y^3 + 2 \Rightarrow x - 2 = y^3 \Rightarrow y = \sqrt[3]{x-2} = f^{-1}(x)$

one-to-one

$(\sqrt[3]{x-2})^3 + 2 = x - 2 + 2 = x$   $D_{f^{-1}}$ : all reals

c.  $f(x) = \frac{2x+1}{x-3} \Rightarrow y = \frac{2x+1}{x-3} \Rightarrow x = \frac{2y+1}{y-3} \Rightarrow xy - 3x = 2y + 1 \Rightarrow$

one-to-one

$xy - 2y = 3x + 1 \Rightarrow y(x-2) = 3x + 1 \Rightarrow y = \frac{3x+1}{x-2}$

domain  $f^{-1}$ : all reals  $\neq 2$

$\frac{2\left(\frac{3x+1}{x-2}\right) + 1}{\left(\frac{3x+1}{x-2}\right) - 3} \cdot \frac{x-2}{x-2} = \frac{6x+2+x-2}{3x+1-3x+6} = \frac{7x}{7} = x$

d.  $f(x) = \sqrt[3]{x} + 1 \Rightarrow y = \sqrt[3]{x} + 1 \Rightarrow x = \sqrt[3]{y} + 1 \Rightarrow x - 1 = \sqrt[3]{y} \Rightarrow (x-1)^3 = y$

one-to-one

domain  $f^{-1}$ : all reals  $y = \sqrt[3]{(x-1)^3} + 1 = x - 1 + 1 = x$

e.  $f(x) = e^{2x+1} \Rightarrow y = e^{2x+1} \Rightarrow x = \frac{1}{2} \ln y \Rightarrow \ln x = 2y + 1 \Rightarrow (\ln x) - 1 = 2y$

one-to-one

$y = \frac{1}{2} [\ln x - 1]$  domain  $f^{-1}$ :  $(0, \infty)$

$x = e^{\frac{1}{2}(\ln x - 1) + 1} = e^{\ln x - 1 + 1} = e^{\ln x} = x$

f. one-to-one

domain  $f^{-1} = [0, 1]$

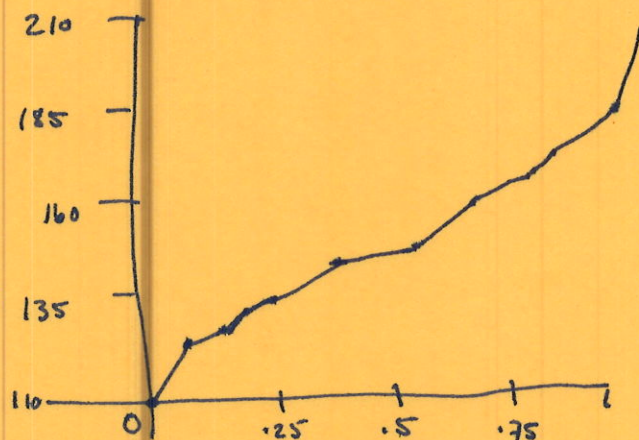
g.  $f(x) = \frac{3}{x-4} \Rightarrow y = \frac{3}{x-4} \Rightarrow x = \frac{3}{y-4}$

one-to-one

$\Rightarrow xy - 4x = 3 \Rightarrow \frac{xy}{x} = \frac{3+4x}{x} \Rightarrow f^{-1} = \frac{3+4x}{x}$

$D_{f^{-1}}$ :  $x \neq 0$

$\frac{3}{\left(\frac{3+4x}{x}\right) - 4} \cdot \frac{x}{x} = \frac{3x}{3+4x-4x} = \frac{3x}{3} = x$



1. h.  $f(x) = \sqrt{x} \Rightarrow y = \sqrt{x} \Rightarrow x^2 = y$   $Df' = x > 0$

one-to-one  $\Rightarrow x = \sqrt{y}$

$y = (\sqrt{x})^2 = x$

i.  $f(x) = x^2 - 4 \Rightarrow y = x^2 - 4 \Rightarrow x = \sqrt{y + 4}$  restrict domain to  $x > 0$

$x + 4 = y^2 \Rightarrow \sqrt{x + 4} = y$  Domain  $f^{-1}: x > -4$

$(\sqrt{x + 4})^2 - 4 = x + 4 - 4 = x$

j.  $f(x) = x^2 + x + 2 \Rightarrow y = x^2 + x + 2 = (x + 1/2)^2 + 7/4$  restrict domain to  $x > -1/2$

$x = (y + 1/2)^2 + 7/4 \Rightarrow x - 7/4 = (y + 1/2)^2 \Rightarrow$

$\sqrt{(x - 7/4)} = y + 1/2 \Rightarrow y = \sqrt{x - 7/4} - 1/2$  domain  $x > 7/4$   
 $f^{-1}$

$(\sqrt{x - 7/4} - 1/2)^2 + \sqrt{x - 7/4} + 1/2 + 2 = x - 7/4 - \sqrt{x - 7/4} + 1/4 + \sqrt{x - 7/4} - 1/2 + 2$

$= x - 7/4 - 1/4 + 2 = x - 2 + 2 = x$

k.  $f(x) = 3 \ln(x + 1) \Rightarrow y = 3 \ln(x + 1) \Rightarrow x = 3 \ln(y + 1) \Rightarrow \frac{x}{3} = \ln(y + 1) \Rightarrow$

one-to-one  $e^{x/3} = y + 1 \Rightarrow e^{x/3} - 1 = y$  D: all reals  
 $f^{-1}$

$3 \ln(e^{x/3} - 1 + 1) = 3 \ln(e^{x/3}) = 3(x/3) = x$

2a.  $\frac{3(x+h) + 7 - (3x + 7)}{h} = \frac{3x + 3h + 7 - 3x - 7}{h} = \frac{3h}{h} = h$

b.  $\frac{(x+h)^2 - 4(x+h) + 3 - (x^2 - 4x + 3)}{h} = \frac{x^2 + 2xh + h^2 - 4x - 4h + 3 - x^2 + 4x - 3}{h} = \frac{h(2x + h - 4)}{h} = 2x + h - 4$

c.  $\frac{b - b}{h} = 0$

d.  $\frac{(\sqrt{x+h-1} - \sqrt{x-1})(\sqrt{x+h-1} + \sqrt{x-1})}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{x+h-1 - (x-1)}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{x+h-1 - x+1}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{h}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{1}{\sqrt{x+h-1} + \sqrt{x-1}}$