

MAT 142 Homework #8 Key

(1)

a. $\tan x \csc x \cos x = 1$ (prove)

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \cos x = 1$$

b. $\frac{\cos \theta \sec \theta}{\cot \theta} = \frac{\cancel{\cos \theta} \cdot \frac{1}{\cancel{\cos \theta}}}{\cot \theta} = \frac{1}{\cot \theta} = \tan \theta$

c. $\frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \frac{\sin t}{\left(\frac{\sin t}{\cos t}\right)} + \frac{\cos t}{\left(\frac{\cos t}{\sin t}\right)} = \sin t \left(\frac{\cos t}{\sin t}\right) + \cos t \left(\frac{\sin t}{\cos t}\right) = \cos t + \sin t$

d. $\frac{\sec x - \csc x}{\sec x + \csc x} \cdot \frac{\sin x + \cos x}{\sin x + \cos x} = \frac{\frac{\sin x}{\cos x} + 1 - 1 + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + 1 + 1 + \frac{\cos x}{\sin x}} =$

$$\frac{\tan x - \cot x}{\tan x + 2 + \cot x} \cdot \frac{\tan x}{\tan x} = \frac{\tan^2 x - 1}{\tan^2 x + 2 \tan x + 1} = \frac{(\cancel{\tan x + 1})(\tan x - 1)}{(\cancel{\tan x + 1})(\tan x + 1)}$$

$$= \frac{\tan x - 1}{\tan x + 1}$$

e. $\frac{1 + \cos t}{1 - \cos t} \cdot \frac{\frac{1}{\sin t}}{\frac{1}{\sin t}} = \frac{\csc t + \cot t}{\csc t - \cot t} \cdot \frac{\csc t + \cot t}{\csc t + \cot t} = \frac{(\csc t + \cot t)^2}{\csc^2 t - \cot^2 t} = 1$

$$= (\csc t + \cot t)^2$$

f. $(\tan^2 \theta + 1)(\cos^2 t + 1) = \frac{\sin^2 t}{\cos^2 t} \cos^2 t + \tan^2 t + \cos^2 t + 1 = \sin^2 t + \cos^2 t + \tan^2 t + 1 = 1 + \tan^2 t + 1 = \tan^2 t + 2$