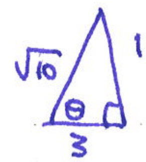
MAT 142, Quiz #12, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round.

1. For $\cot\theta = 3$, and θ in Q III, find:
   a. $\tan 2\theta$

$\tan\theta = \frac{1}{3}$

$$\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2} = \frac{\frac{2}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{\cancel{2}^{1}}{\cancel{3}} \cdot \frac{\cancel{9}^{3}}{\cancel{8}_{4}} = \frac{3}{4}$$



   b. $\cos\frac{\theta}{2}$

$\frac{\theta}{2}$ in QII

$$-\sqrt{\frac{1+\frac{3}{\sqrt{10}}}{2}} = -\sqrt{\frac{\sqrt{10}+3}{2\sqrt{10}}}$$

2. Verify the identities.
   a. $\cot x = \frac{\sin 2x}{1-\cos 2x}$

$$\frac{\cos x}{\sin x} \cdot \frac{2\sin x}{2\sin x} = \frac{2\sin x\cos x}{2\sin^2 x} = \frac{\sin 2x}{1-\cos 2x}$$

$$\cos 2x = 1 - 2\sin^2 x$$

   b. $\cos 4t = 8\cos^4 t - 8\cos^2 t + 1$

$2\cos^2 2t - 1$

$2(2\cos^2 t - 1)^2 - 1$

$2(4\cos^4 t - 4\cos^2 t + 1) - 1$

$8\cos^4 t - 8\cos^2 t + 2 - 1$

$8\cos^4 t - 8\cos^2 t + 1$

Some useful formulas:

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\begin{aligned} \sin 2\alpha &= 2 \sin \alpha \cos \alpha \\ \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= 2\cos^2 \alpha - 1 \\ &= 1 - 2\sin^2 \alpha \\ \tan 2\alpha &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \end{aligned}$$

$$\sin(\frac{a}{2}) = \pm\sqrt{\frac{(1-\cos a)}{2}}$$

$$\cos(\frac{a}{2}) = \pm\sqrt{\frac{(1+\cos a)}{2}}$$

$$\tan(\frac{a}{2}) = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a}$$

$$\sin^2 \theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2 \theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$