MAT 212, Exam #2, Spring 2017 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Solve the second-order ODEs for the general solution. (9 points each)
   a. $2y'' - y' - y = 0$

$2r^2 - r - 1 = 0$

$(2r+1)(r-1) = 0$

$r = -\frac{1}{2}, r = 1$

$y = c_1 e^{-1/2 t} + c_2 e^{t}$

   b. $y'' - 2y' + 2y = 0$

$r^2 - 2r + 2 = 0$

$r = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$

$y = c_1 e^{t} \cos t + c_2 e^{t} \sin t$

   c. $x^2y'' + 2xy' - 6y = 0$

$n(n-1) + 2n - 6 = 0$

$n^2 - n + 2n - 6 = 0$

$n^2 + n - 6 = 0$

$(n+3)(n-2) = 0$

$n = -3, n = 2$

$y = c_1 t^{-3} + c_2 t^{2}$

d. $y'' - 18y' + 81y = 0$

$r^2 - 18r + 81 = 0$

$(r-9)^2 = 0$

$r = 9$ repeated

$y = c_1 e^{9t} + c_2 t e^{9t}$

e. $y''' + 2y'' - y' - 2y = 0$

$r^3 + 2r^2 - r - 2 = 0$

$r^2(r+2) - 1(r+2) = 0$

$(r^2-1)(r+2) = 0$

$(r+1)(r-1)(r+2) = 0$

$r = -1,\ r = 1,\ r = -2$

$y = c_1 e^{-t} + c_2 e^{t} + c_3 e^{-2t}$

2. The table below gives the solution to the second order constant coefficient homogeneous equation, and the forcing function $F(x)$ or $F(t)$. Determine the Ansatz for the method of undetermined coefficients in each case. (4 points each)

	y_1	y_2	y_3	$F(x)$ or $F(t)$	Ansatz
a.	e^{-2x}	e^{3x}	NA	$2\sin 3x$	$A\cos 3x + B\sin 3x$
b.	$e^{-x}\cos x$	$e^{-x}\sin x$	NA	$e^{x}\sin x$	$Ae^{x}\cos x + Be^{x}\sin x$
c.	e^{x}	$e^{-x/2}\cos\left(\frac{\sqrt{3}}{2}x\right)$	$e^{-x/2}\sin\left(\frac{\sqrt{3}}{2}x\right)$	$e^{x}+7$	$Axe^{x} + B$
d.	t	1	e^{-t}	$t + e^{-t}$	$At^3 + Bt^2 + Cte^{-t}$
e.	$\sin t$	$\cos t$	e^{-t}	$\cos^2 t$	$A + B\cos 2t + C\sin 2t$

$= \frac{1}{2}(1 + \cos 2t)$

3. What is the difference between the natural frequency of the system, and a quasi-frequency? How is each obtained? (4 points)

The natural frequency is what the frequency is when there is no damping. The quasi-frequency depends on the damping. The natural frequency is $\omega=\sqrt{\frac{k}{m}}$. The quasi-frequency is $\frac{\sqrt{\gamma^2-4km}}{2m}=\omega$.

4. What conditions are needed in a forced oscillation system to achieve beats? (4 points)

The system needs to be undamped and the forcing to the system needs to be similar to, but not the same as, the natural frequency.

5. Use the method of reduction of order to solve $(1-x^2)y''-2xy'+2y=0$, given $y_1(x)=x$. (12 points)

$y_2 = v \cdot y_1$ $\quad y_1' = 1$

$y_2' = v'y_1 + vy_1'$ $\quad y_1''=0$

$y_2'' = v''y_1 + 2v'y_1' + \cancel{y_1''}$

$y_2 = vx$

$y_2' = v'x + v$

$y_2'' = v''x + 2v'$

$(1-x^2)(v''x+2v') - 2x(v'x+v) + 2vx = 0$

$xv''+2v'-v''x^3-2v'x^2-2x^2v'-\cancel{2vx}+\cancel{2vx}=0$

$v''(x-x^3)+2v'(1-2x^2)$

$xv''(1-x^2) = -2v'(1-2x^2)$

$v'' = \frac{-2(1-2x^2)}{x(1-x^2)}v'$

let $u=v'$

$v''=\frac{du}{dx}$

$\frac{A}{x}+\frac{B}{1-x}+\frac{C}{1+x}=\frac{-2+4x^2}{\sim}$

$A-Ax^2+Bx+Bx^2+Cx+Cx^2=-2+4x^2$

$-Ax^2+Bx^2-Cx^2=4x^2$

$A=-2$

$Bx+Cx=0x$ $\quad B+C=0 \Rightarrow B=-C$

$-2+B+B=4$ $\quad 2B=6$ $\quad B=3,\ C=-3$

$v=\int \frac{1}{x^2(1-x)^3(1+x)^3}dx$

$\frac{du}{dx} = \frac{-2+4x^2}{x(1-x)(1+x)}u \Rightarrow \int\frac{du}{u} = \int\frac{-2+4x^2}{x(1-x)(1+x)}dx$

$y_2 = x\int\frac{1}{x^2(1-x)^3(1+x)^3}dx$

$=\int \frac{-2}{x}+\frac{3}{1-x}-\frac{3}{1+x}dx \Rightarrow \ln u = -2\ln x - 3\ln(1-x) - 3\ln(1+x)$

$u = x^{-2}(1-x)^{-3}(1+x)^{-3}$