

Instructions: Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Use the definition of the Laplace transform $F(s) = \int_0^{\infty} e^{-st} f(t) dt$ to find $\mathcal{L}\{t - 2e^{3t}\}$.

$$\int_0^{\infty} e^{-st} (t - 2e^{3t}) dt = \int_0^{\infty} te^{-st} - 2e^{-t(s+3)} dt$$

$$-t\left(\frac{1}{s}\right)e^{-st} - \frac{1}{s^2}e^{-st} - \frac{2e^{-t(s+3)}}{s+3} \Big|_0^{\infty}$$

$$\left(0 - \frac{1}{s^2}(0) - 0\right) - \left(0 - \frac{1}{s^2} - \frac{2}{s+3}\right) =$$

$$\frac{1}{s^2} + \frac{2}{s+3}$$

\pm	u	dv
+	t	e^{-st}
-	1	$-\frac{1}{s}e^{-st}$
+	0	$\frac{1}{s^2}e^{-st}$

2. Use the attached table of Laplace transforms to find:

a. $\mathcal{L}\{\sin 2t - \cos 3t\}$

$$\frac{2}{s^2+4} - \frac{s}{s^2+9}$$

b. $\mathcal{L}\{t^{3/2} - e^{-10t}\}$

$$\frac{\Gamma(\frac{3}{2}+1)}{s^{5/2}} - \frac{1}{s+10} = \frac{15\sqrt{\pi}}{8s^{5/2}} - \frac{1}{s+10}$$

c. $\mathcal{L}^{-1}\left\{9 + \frac{s}{4-s^2}\right\}$

$$9 - \frac{s}{s^2-4}$$

$$9\delta(t) - \cosh(2t)$$

d. $\mathcal{L}^{-1}\left\{\frac{2e^{-3s}}{s}\right\}$

$$2u(t-3)$$