MAT 212, Quiz #12, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Solve $y'' + y = \sin 2t, y(0) = y'(0) = 0$ using Laplace transforms.

$s^2 F(s) - s(0) - (0) + F(s) = \frac{2}{s^2+4}$

$F(s)(s^2 + 1) = \frac{2}{s^2+4}$

$F(s) = \frac{2}{(s^2+4)(s^2+1)}$

$\frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+1} = \frac{2}{(s^2+4)(s^2+1)}$

$As^3 + Bs^2 + As + B + Cs^3 + Ds^2 + 4Cs + 4D = 2$

$(A+C) = 0$ $\quad A + 4C = 0$ $\quad A = -C$

$(B+D) = 0$ $\quad B + 4D = 2$ $\quad A - 4A = 0$

$B = -D$ $\quad B - 4B = 2$ $\quad A = 0$, $C = 0$

$-3B = 2 \Rightarrow B = -\frac{2}{3}$

$D = \frac{2}{3}$

$F(s) = \frac{-2/3}{s^2+4} + \frac{2/3}{s^2+1}$

$Y(x) = -\frac{2}{3}\sin 2t + \frac{2}{3}\sin t$

2. Use the attached table of Laplace transforms to find:

a. $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2-9)}\right\}$ $\quad \frac{1}{s} \cdot \frac{1}{s^2-9}$ $\quad \int_0^t (t-\tau) \sinh 3\tau \, d\tau$

b. $\mathcal{L}^{-1}\left\{\frac{3s+5}{s^2-6s+25}\right\}$ $\quad 3(s-3)+14$ $\quad \frac{3(s-3)}{(s-3)^2+4^2} + \frac{14}{(s-3)^2+4^2} \Rightarrow 3e^{3t}\cos 4t + \frac{7}{2}e^{3t}\sin 4t$

$(s^2-6s+9)+16$

$(s-3)^2+4^2$

c. $\mathcal{L}^{-1}\left\{\frac{s^2}{(s^2+4)^2}\right\}$ $\quad \frac{1}{4}[\sin 2t + 2t\cos 2t] = \frac{1}{4}\sin 2t + \frac{1}{2}t\cos 2t$

d. $\mathcal{L}^{-1}\left\{\frac{s}{(s+3)(s^2-1)}\right\}$ $\quad \frac{1}{s+3} \cdot \frac{s}{s^2-1} \Rightarrow \int_0^t e^{-3(t-\tau)} \cosh \tau \, d\tau$

e. $\mathcal{L}^{-1}\left\{\frac{s(1-e^{-2s})}{s^2+\pi^2}\right\} = \frac{s}{s^2+\pi^2} - e^{-2s}\left(\frac{s}{s^2+\pi^2}\right)$

$\cos \pi t - u(t-2)\cos[\pi(t-2)]$