

**Instructions:** Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Write the system  $\begin{cases} x' = 3x - 2y \\ y' = 2x + y \end{cases}$  as a single second-order system.

$$x'' - 4x' + 7x = 0$$

$$\frac{2y}{2} = \frac{3x - x'}{2}$$

$$y = \frac{3}{2}x - \frac{1}{2}x'$$

$$y' = \frac{3}{2}x' - \frac{1}{2}x'' = 2x + \left(\frac{3}{2}x - \frac{1}{2}x'\right)$$

$$\frac{3}{2}x' - \frac{1}{2}x'' = \frac{7}{2}x - \frac{1}{2}x'$$

$$-\frac{1}{2}x'' + 2x' - \frac{7}{2}x = 0 \quad \times(-2)$$

2. Solve the system  $\vec{x}' = \begin{bmatrix} -3 & 2 \\ -3 & 4 \end{bmatrix} \vec{x}$  for the general solution.

$$(-3-\lambda)(4-\lambda) + 6 = 0$$

$$\lambda^2 + 3\lambda - 4\lambda - 12 + 6 = 0$$

$$\lambda^2 - \lambda - 6 = 0$$

$$(\lambda-3)(\lambda+2) = 0$$

$$\lambda = 3, -2$$

$$\lambda_1 = 3$$

$$\begin{bmatrix} -6 & 2 \\ -3 & 1 \end{bmatrix}$$

$$-3x_1 = -x_2$$

$$x_1 = \frac{1}{3}x_2$$

$$x_2 = x_2$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\lambda_2 = -2$$

$$\begin{bmatrix} -1 & 2 \\ -3 & 6 \end{bmatrix}$$

$$-x_1 = -2x_2$$

$$x_1 = 2x_2$$

$$x_1 = x_2$$

$$\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t}$$

3. Solve  $\begin{cases} x_1' = 6x_1 - 7x_2 \\ x_2' = x_1 - x_2 \end{cases}$  and write the fundamental solution matrix.

$$\begin{bmatrix} 6 & -7 \\ 1 & -1 \end{bmatrix}$$

$$(6-\lambda)(-1-\lambda) + 7 = 0$$

$$\lambda^2 - 6\lambda + \lambda - 6 + 7 = 0$$

$$\lambda^2 - 5\lambda + 1 = 0$$

$$\lambda = \frac{5 \pm \sqrt{25-4}}{2} = \frac{5 \pm \sqrt{21}}{2}$$

$$\lambda_1 = \frac{5 + \sqrt{21}}{2}$$

$$\begin{bmatrix} 6 - \frac{5 + \sqrt{21}}{2} & -7 \\ 1 & -1 - \frac{5 + \sqrt{21}}{2} \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} 7 + \sqrt{21} \\ 2 \end{bmatrix}$$

$$\lambda_2 = \frac{5 - \sqrt{21}}{2}$$

$$\vec{v}_2 = \begin{bmatrix} 7 - \sqrt{21} \\ 2 \end{bmatrix}$$

$$\vec{x}(t) = \Psi(t) = \begin{pmatrix} (7 + \sqrt{21})e^{\frac{5 + \sqrt{21}}{2}t} & (7 - \sqrt{21})e^{\frac{5 - \sqrt{21}}{2}t} \\ 2e^{\frac{5 + \sqrt{21}}{2}t} & 2e^{\frac{5 - \sqrt{21}}{2}t} \end{pmatrix}$$