

Instructions: Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Solve the system of equations $\begin{cases} 2x - 3y = 11 \\ 4x + 7y = 18 \end{cases}$ by row-reducing.

$$\left[\begin{array}{cc|c} 2 & -3 & 11 \\ 4 & 7 & 18 \end{array} \right]$$

$$-2R_1 + R_2 \rightarrow R_2$$

$$\left[\begin{array}{cc|c} 2 & -3 & 11 \\ 0 & 13 & -4 \end{array} \right]$$

$$-13R_2 \rightarrow R_2$$

$$\left[\begin{array}{cc|c} 2 & -3 & 11 \\ 0 & 1 & -4/13 \end{array} \right]$$

$$3R_2 + R_1 \rightarrow R_1$$

$$\left[\begin{array}{cc|c} 2 & 0 & 13/13 \\ 0 & 1 & -4/13 \end{array} \right]$$

$$\frac{1}{2}R_1 \rightarrow R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & 13/26 \\ 0 & 1 & -4/13 \end{array} \right]$$

2. Classify the differential equations by i) order, ii) linearity, and iii) ordinary or partial.

a. $(y')^2 + y^2 = 1$

1st order, nonlinear, ordinary

b. $x^2y'' + xy' - y = \ln x$

2nd order, linear, ordinary

c. $u_{xy} + u_{yy} = u_x$

2nd order, linear, partial

3. Solve by separation $\frac{dy}{dx} = 3x^2(y^2 + 1)$

$$\int \frac{dy}{y^2+1} = \int 3x^2 dx$$

$$\arctan y = x^3 + C$$

4. Solve by the method of integrating factors $xy' + 3y = 2x^5$.

$$y' + \frac{3}{x}y = 2x^4$$

$$\mu = e^{\int \frac{3}{x} dx} = e^{3 \ln x} = e^{\ln x^3} = x^3$$

$$x^3 y' + 3x^2 y = 2x^7$$

$$\int (x^3 y)' = \int 2x^7$$

$$x^3 y = \frac{1}{4} x^8 + C$$

$$y = \frac{1}{4} x^5 + \frac{C}{x^3}$$