

Instructions: Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Find the inverse of $\begin{bmatrix} 2 & -3 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} e^t & e^{-2t} \\ 2e^t & 3e^{-2t} \end{bmatrix}$.

$$\frac{1}{14 - (-12)} \begin{bmatrix} 7 & 3 \\ -4 & 2 \end{bmatrix} = \frac{1}{26} \begin{bmatrix} 7 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 7/26 & 3/26 \\ -4/26 & 2/26 \end{bmatrix}$$

$$\frac{1}{3e^{-t} - 2e^{-t}} \begin{bmatrix} 3e^{-2t} & -e^{-2t} \\ -2e^t & e^t \end{bmatrix} = \frac{1}{e^{-t}} \begin{bmatrix} 3e^{-2t} & -e^{-2t} \\ -2e^t & e^t \end{bmatrix} =$$

$$e^t \begin{bmatrix} 3e^{-2t} & -e^{-2t} \\ -2e^t & e^t \end{bmatrix} = \begin{bmatrix} 3e^{-t} & -e^{-t} \\ -2e^{2t} & e^{2t} \end{bmatrix}$$

2. Solve $y'' - 3y' + 2y = 0$, $y(0) = 1$, $y'(0) = 2$.

$$r^2 - 3r + 2 = 0$$

$$(r-2)(r-1) = 0$$

$$r=2, r=1$$

$$y = c_1 e^{2t} + c_2 e^t$$

$$y' = 2c_1 e^{2t} + c_2 e^t$$

$$1 = c_1 + c_2$$

$$2 = 2c_1 + c_2$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 1 & 2 \end{array} \right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \end{array} \right] \quad c_1 = 1, c_2 = 0$$

$$y(t) = e^{2t}$$