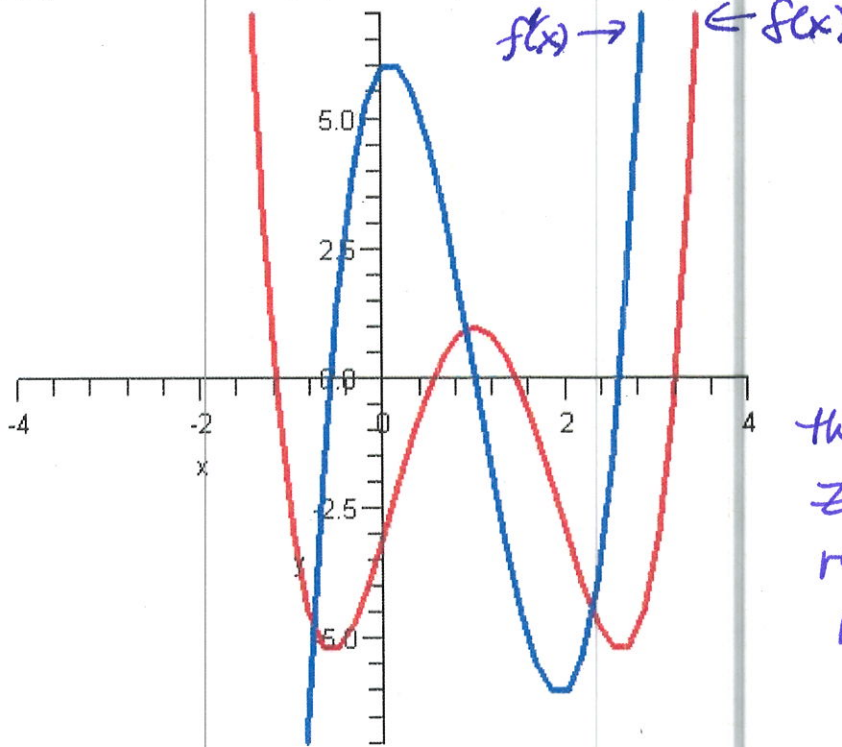


Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. On this portion of the exam, **no calculator is permitted.**

1. The graph below shows two functions. One function $f'(x)$ is the derivative of the other function $f(x)$. Determine which graph is which, and label both. (6 points)

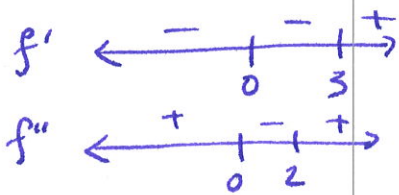


the blue graph is zero whenever the red graph has a max or min.

2. Consider the function $f(x) = x^4 - 4x^3 + 10$. Find all the extrema and inflection points. Classify all extrema. Use that information to sketch the curve. (15 points)

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3) \quad x=0, x=3$$

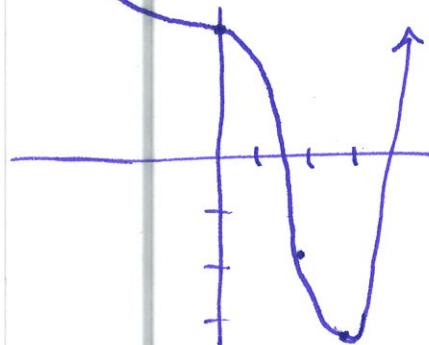
$$f''(x) = 12x^2 - 24x = 12x(x-2) \quad x=0, x=2$$



$$f(0) = 10$$

$$f(3) = -17$$

$$f(2) = -6$$



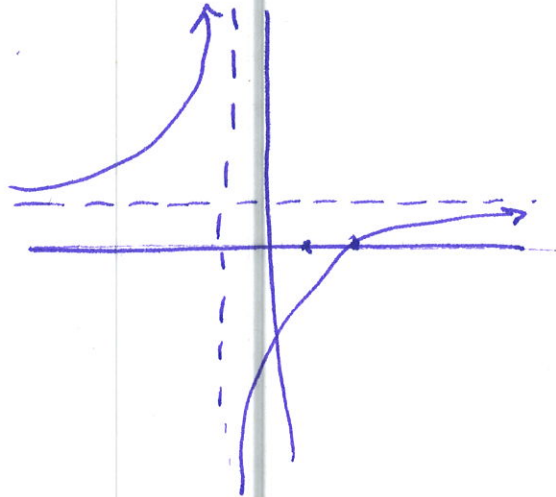
3. Find all asymptotes, intercepts and critical points, and use that information to graph $f(x) = \frac{x-2}{x+1}$. (10 points)

vertical at $x = -1$
 x-intercept at $x = 2$
 horizontal at $y = 1$

$$y' = \frac{1(x+1) - (x-2)(1)}{(x+1)^2}$$

$$= \frac{x+1-x+2}{(x+1)^2} = \frac{3}{(x+1)^2}$$

always positive
 always increasing
 (except $x = -1$)



4. Find $\frac{dy}{dx}$ for $3x^2y^4 = 12$. (8 points)

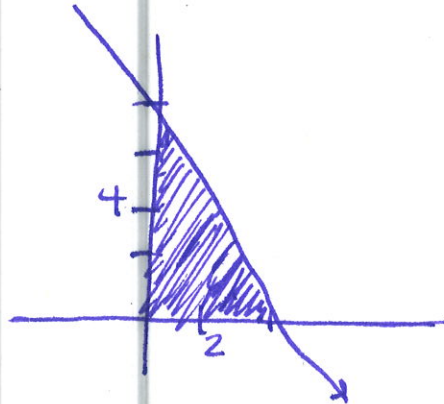
$$6xy^4 + 12x^2y^3y' = 0$$

$$12x^2y^3y' = -6xy^4$$

$$y' = \frac{-6xy^4}{12x^2y^3} = \frac{-y}{2x}$$

5. Geometrically find the area under the curve $y = 4 - 2x$ on $[0, 2]$. [Hint: Sketch the graph. What shape is this?] (8 points)

$$A = \frac{1}{2}bh = \frac{1}{2}(2)(4) = 4$$



6. Evaluate $\int_{-2}^3 -x^2 + 4x - 5 dx$. (8 points)

$$-\frac{1}{3}x^3 + 2x^2 - 5x \Big|_{-2}^3 =$$

$$-\frac{1}{3}(27) + 2(9) - 5(3) - \left[-\frac{1}{3}(-8) + 2(4) - 5(-2) \right]$$

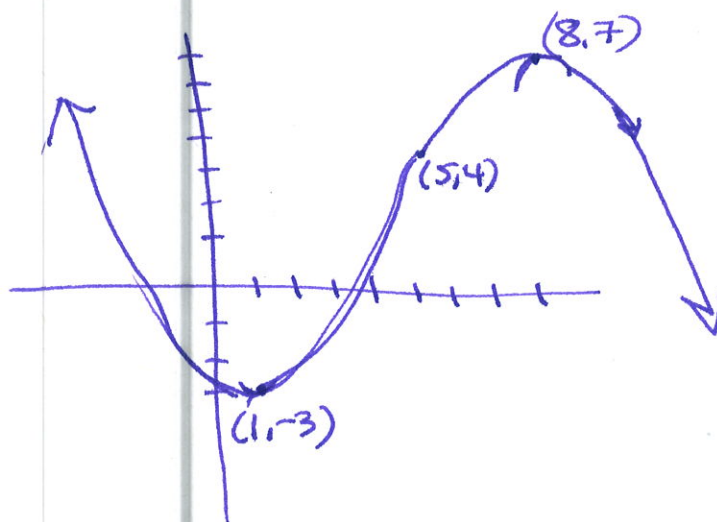
$$= -9 + 18 - 15 - \left[\frac{8}{3} + 8 + 10 \right] =$$

$$-6 - \left[\frac{62}{3} \right] = -\frac{80}{3}$$



Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. You may use a calculator on this portion of the exam. If you use your calculator, describe the steps you used, or sketch the graph obtained from your calculator to show work.

- Sketch the graph of a function that is concave up at $(1, -3)$, concave down at $(8, 7)$, and has an inflection point at $(5, 4)$. (6 points)



- Find the absolute extrema of the function on the given interval. (5 points each)

a. $f(x) = 24, [4, 13]$

function is constant. every point on the interval is the same and both the max & min

$$f'(x) = 0$$

b. $f(x) = 1 - x^{2/3}, [-8, 8]$

$$f' = -\frac{2}{3}x^{-1/3} = 0 ?$$

never zero

$-\frac{2}{3\sqrt[3]{x}}$ undefined at $x=0$
critical point

test $-8, 8, 0$

$$f(-8) = -3 \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{also min}$$

$$f(8) = -3$$

$$f(0) = 1 \leftarrow \text{abs max}$$

c. $f(x) = x + \frac{1}{x}, [1, 20]$ $f'(x) = 1 - \frac{1}{x^2} = 0$ $x = \pm 1$
 -1 not in interval

Check 1, 20

$f(1) = 2 \leftarrow$ abs min

$f(20) = 20.05 \leftarrow$ abs max

3. If the revenue function for a product is $R(x) = 50x - \frac{1}{2}x^2$, and the cost function is $C(x) = 4x + 10$, maximize the profit. (9 points)

profit = Rev - cost = $50x - \frac{1}{2}x^2 - (4x + 10) =$

$50x - \frac{1}{2}x^2 - 4x - 10 =$

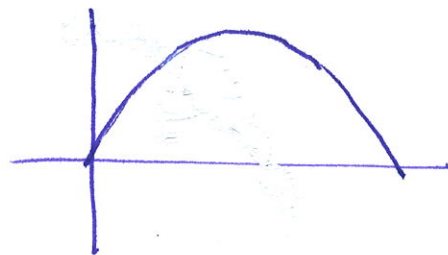
$-\frac{1}{2}x^2 + 46x - 10$

Profit

$P' = -x + 46$

$x = 46$

max profit. $P(46) = 1048$



4. Suppose the price p and sales x of a mechanical pencil are related by the equation $5p + 4x + 2px = 60$. Find the rate of change of sales when $x = 3, p = 5, \frac{dp}{dt} = 1.5$. (8 points)

$5 \frac{dp}{dt} + 4 \frac{dx}{dt} + 2 \frac{dp}{dt} x + 2p \frac{dx}{dt} = 0$

$5(1.5) + 4 \left(\frac{dx}{dt} \right) + 2(1.5)(3) + 2(5) \left(\frac{dx}{dt} \right) = 0$

$7.5 + [4 + 10] \frac{dx}{dt} + 9 = 0$

$14 \frac{dx}{dt} = -16.5$

$\frac{dx}{dt} = -\frac{33}{28}$