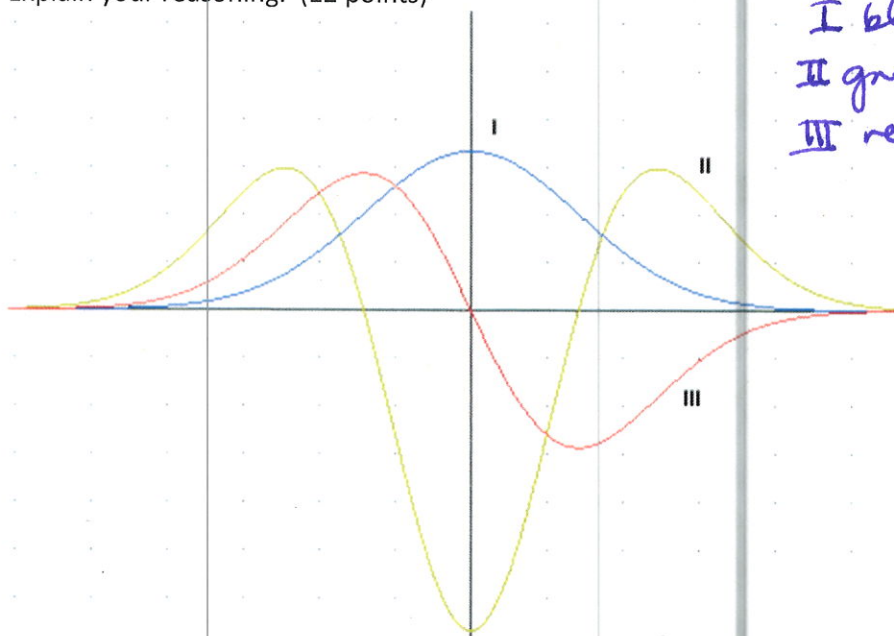


**Instructions:** Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. On this portion of the exam, **no calculator is permitted.**

- The graph below shows three functions. One function  $f'(x)$  is the derivative of the other function  $f(x)$ , and the third function is  $f''(x)$ . Determine which graph is which, and label each. Explain your reasoning. (12 points)

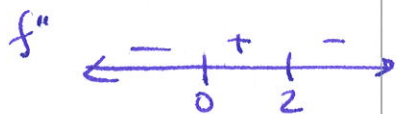


I blue line is  $f(x)$   
 II green line is  $f''(x)$   
 III red line is  $f'(x)$

- Consider the function  $f(x) = x^4 - 4x^3 + 10$ . Find all the extrema and inflection points. Classify all extrema. Use that information to sketch the curve. (15 points)

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3) \quad x=0, x=3$$

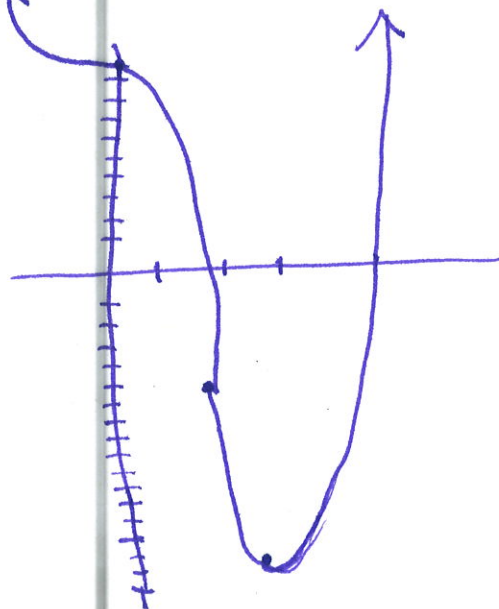
$$f''(x) = 12x^2 - 24x = 12x(x-2) \quad x=0, x=2$$



$$f(0) = 10$$

$$f(2) = -6$$

$$f(3) = -17$$



3. Find  $\frac{dy}{dx}$  for  $y^5\sqrt{x} = 96$ . Use it to find the slope of the tangent line at (9,2). (15 points)

$$y^5 x^{1/2} = 96$$

$$5y^4 y' x^{1/2} + y^5 \cdot \frac{1}{2} x^{-1/2} = 0$$

$$5y^4 x^{1/2} y' = -\frac{1}{2} y^5 x^{-1/2}$$

$$y' = \frac{-\frac{1}{2} y^5 x^{-1/2}}{5y^4 x^{1/2}} = \frac{-y}{10x}$$

$$\text{Slope} = \frac{-2}{90} = -\frac{1}{45}$$

$$y - 2 = -\frac{1}{45}(x - 9) \Rightarrow$$

$$y - 2 = -\frac{1}{45}x + \frac{1}{5} \Rightarrow y = -\frac{1}{45}x + \frac{11}{5}$$

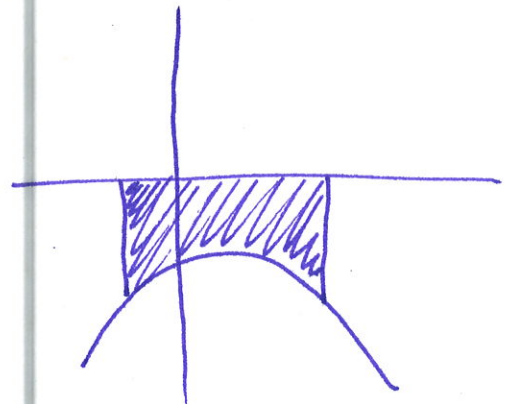
4. Evaluate  $\int_{-2}^3 -3x^2 + 4x - 7 \, dx$ . (12 points)

$$\left. -\frac{3}{3}x^3 + 2x^2 - 7x \right|_{-2}^3$$

$$-(3)^3 + 2(9) - 7(3) - [ -(-2)^3 + 2(-2)^2 - 7(-2) ] =$$

$$-27 + 18 - 21 - [ 8 + 8 + 14 ] =$$

$$-30 - [ 30 ] = -60$$



5. Evaluate  $f(x, y, z) = 2^x + 5zy - x$  at (0,1,-3) and (1,0,-3). (8 points)

$$f(0,1,-3) = 2^0 + 5(-3)(1) - 0 = 1 - 15 - 0 = -14$$

$$f(1,0,-3) = 2^1 + 5(-3)(0) - 1 = 2 - 0 - 1 = 1$$

6. Find the domain of the functions. (7 points each)

a.  $g(x, y) = \ln(x - y)$

$$x - y > 0$$

$$x > y$$

$$D: \{(x, y) \mid x > y\}$$

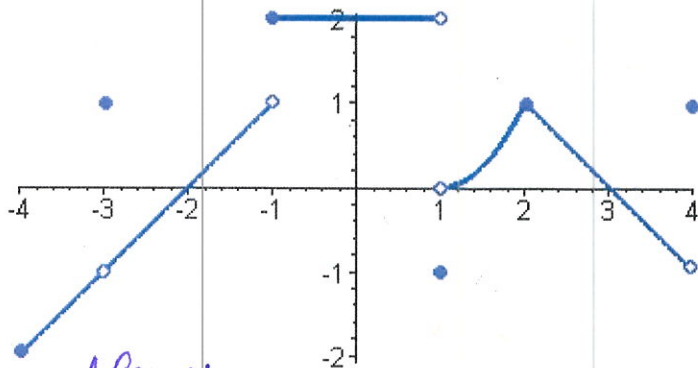
b.  $g(x, y) = \frac{1}{y + x^2}$

$$y + x^2 \neq 0$$

$$x^2 \neq -y$$

$$D: \{(x, y) \mid y \neq -x^2\}$$

7. Consider the graph of the piecewise function  $f(x)$  below. Describe all points of discontinuity (explain your reasoning). Note any places where the graph is continuous but not differentiable. (12 points)



Continuity problems:

hole at  $x = -3$  limit exists but  $\neq f(-3)$

jump at  $x = -1$ , limit does not exist

jump at  $x = 1$ , limit does not exist

hole at  $x = 4$  left hand limit not equal to  $f(4)$

Continuous at all other points in the domain  $[-4, 4]$

8. Find the limit  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$ . Use any method (algebra or L'Hôpital's). (10 points)

$$\frac{0}{0}$$
$$= \lim_{x \rightarrow 1} \frac{3x^2}{1} = 3$$

9. Use the limit definition of the derivative to find  $f'(x)$  for  $f(x) = 3x - x^2$ . (15 points)

$$\lim_{h \rightarrow 0} \frac{3(x+h) - (x+h)^2 - (3x - x^2)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{3x + 3h - x^2 - 2xh - h^2 - 3x + x^2}{h} =$$

$$\lim_{h \rightarrow 0} \frac{3h - 2xh - h^2}{h} = \lim_{h \rightarrow 0} \frac{h(3 - 2x - h)}{h}$$

$$= \lim_{h \rightarrow 0} 3 - 2x - h = 3 - 2x$$

**Instructions:** Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. You may use a calculator on this portion of the exam. If you use your calculator, describe the steps you used, or sketch the graph obtained from your calculator to show work.

1. Find  $f_x$  and  $f_y$  for  $f(x, y) = e^{xy}$ . (10 points)

$$f_x = ye^{xy}$$

$$f_y = xe^{xy}$$

2. Find all extrema for  $f(x, y) = 4xy - x^3 - 2y^2$  and classify all the points as maxima, minima, or saddle points (or say why it's not possible). (20 points)

$$f_x = 4y - 3x^2 = 0 \Rightarrow 4y = 3x^2$$

$$f_y = 4x - 4y = 0 \Rightarrow 4x = 4y \Rightarrow x = y$$

$$4x = 3x^2 \Rightarrow 3x^2 - 4x = 0$$

$$x(3x - 4) = 0$$

$$x = 0, x = \frac{4}{3}$$

$$y = 0, y = \frac{4}{3}$$

$$f_{xx} = -6x$$

$$f_{yy} = -4$$

$$f_{xy} = 4$$

$$D(0, 0) = (0)(-4) - 4^2 = -16 \text{ Saddle point}$$

$$D\left(\frac{4}{3}, \frac{4}{3}\right) = -6\left(\frac{4}{3}\right)(-4) - 16 = 16 \quad f_{xx}, f_{yy} < 0 \text{ max.}$$