

MAT 230 Written Homework #10

(1)

a.  $f(x,y) = x^2 + xy + 3y^2 + 11x$

$f_x = 2x + y + 11 = 0$

$f_y = x + 6y = 0 \Rightarrow x = -6y \Rightarrow 2(-6y) + y + 11 = 0$

$-12y + y = -11 \Rightarrow -11y = -11$

$y = 1$   
 $x = -6$

$(-6, 1)$

$f_{xx} = 2$

$D = 2(6) - 1^2 = 12 - 1 = 11 > 0$

$f_{yy} = 6$

$f_{xx} \cup$

minimum @  $(-6, 1)$

$f_{xy} = 1$

b.  $f(x,y) = x^3 + y^3 - 6xy$

$f_x = 3x^2 - 6y = 0 \Rightarrow 3(\frac{y^2}{2})^2 - 6y = 0 \Rightarrow (\frac{3}{4}y^4 - 6y = 0) \frac{4}{3} \Rightarrow y^4 - 8y = 0$

$f_y = 3y^2 - 6x = 0 \Rightarrow 3y^2 = 6x \Rightarrow y^2 = 2x \Rightarrow \frac{1}{2}y^2 = x$

$y(y^3 - 8) = 0$

$f_{xx} = 6x$

$y = 0 \Rightarrow x = 0$

$y(y-2)(y^2+2y+4) = 0$

$f_{yy} = 6y$

$y = 2 \Rightarrow x = 2$

$y = 0, y = 2$

$f_{xy} = -6$

(others complex)

$D(0,0) = 0(0) - (-6)^2 = -36$  saddle point @  $(0,0)$

$D(2,2) = (12)(12) - (-6)^2 = 144 - 36 > 0$  minimum @  $(2,2)$

$f_{xx} \cup$

c.  $f(x,y) = 4xy - x^3 - 2y^2$

$f_x = 4y - 3x^2 = 0 \Rightarrow 4x - 3x^2 = 0 \Rightarrow x(4 - 3x) = 0$

$f_y = 4x - 4y = 0 \Rightarrow x = y$

$x = 0, x = \frac{4}{3}$

$f_{xx} = -6x$

$y = x$

$f_{yy} = -4$

$(0,0), (\frac{4}{3}, \frac{4}{3})$

$f_{xy} = 4$

$D(0,0) = (0)(-4) - 4^2 = -16 < 0$  saddle point @  $(0,0)$

$D(\frac{4}{3}, \frac{4}{3}) = (-8)(-4) - 4^2 = 32 - 16 = 16 > 0$  maximum @  $(\frac{4}{3}, \frac{4}{3})$

$f_{xx} \cap$

$$2. P(a, n) = -5a^2 - 3n^2 + 48a - 4n + 2an + 290$$

(2)

$$P_a = -10a + 48 + 2n = 0$$

$$P_n = -6n - 4 + 2a = 0 \Rightarrow \frac{2a}{2} = \frac{6n+4}{2} \Rightarrow a = 3n+2$$

a in millions  
n in thousands

$$-10(3n+2) + 48 + 2n = -30n - 20 + 48 + 2n = -28n + 28 = 0$$

$$28 = 28n \Rightarrow n = 1$$

$$a = 5$$

$$P(1, 5) = 248 \text{ (in millions)}$$

3a. linear

$$y = 1.02857x - .82857$$

$$r^2 = .96$$

b. quadratic

$$y = 34x^2 - 100.2x + 78$$

$$R^2 = .99$$

c.  $y = 7.695(1.064)^x$

$$r^2 = .992$$

d.  $y = 5.55 + 8.67 \ln x$

$$r^2 = .84$$

e.  $y = .21x^{3.87}$

$$r^2 = .98$$

