

MAT 230 Written Homework #5 Key

(1)

1. $f(x) = 9x - 3x^2 - x^3$

$x(9 - 3x - x^2) = 0$

$-x(x^2 + 3x - 9) = 0 \quad \frac{-3 \pm \sqrt{9 + 36}}{2} = \frac{-3 \pm \sqrt{45}}{2} = \frac{-3 \pm 3\sqrt{5}}{2} = x$

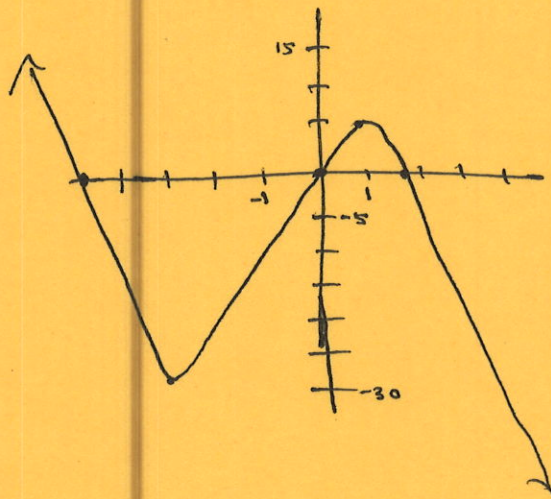
intercepts: $x = 0, x = \frac{-3 + 3\sqrt{5}}{2}, \frac{-3 - 3\sqrt{5}}{2}$

$f'(x) = 9 - 6x - 3x^2$

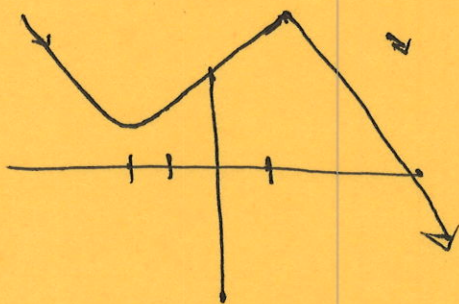
$-3(x^2 + 2x - 3) = 0$

$-3(x+3)(x-1) = 0 \quad x = -3, x = 1$

$f'(-3) = -27 \quad f'(1) = 5$

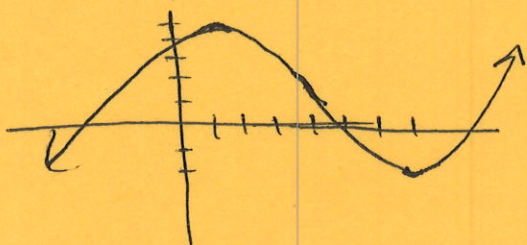


2. a.

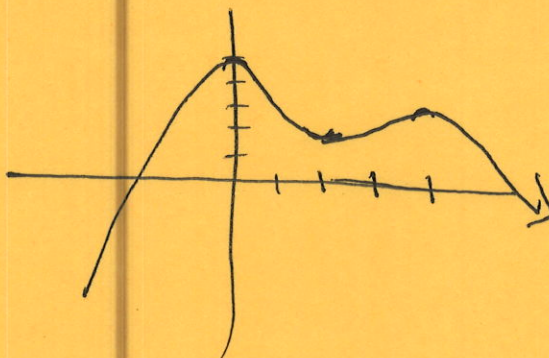


Answers will vary

b.

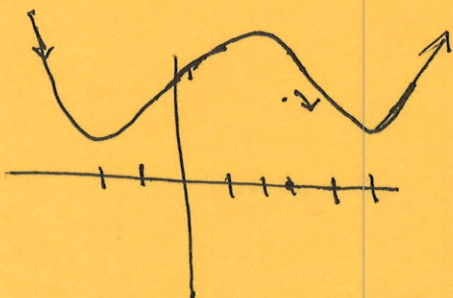


c.

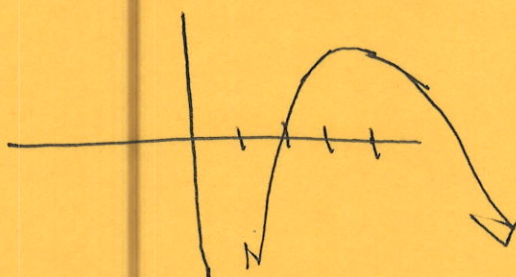


3.

a.



b.



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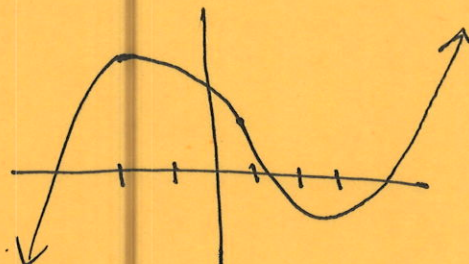
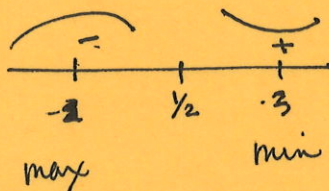
(2)

4. a. $f(x) = 2x^3 - 3x^2 - 36x + 28$

$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2) = 0$

$x = 3, x = -2$

$f''(x) = 12x - 6 = 6(2x - 1) = 0 \quad x = \frac{1}{2}$



b. $f(x) = \frac{8x}{x^2+1}$

$f'(x) = \frac{8(x^2+1) - 8x(2x)}{(x^2+1)^2} = \frac{8x^2+8-16x^2}{(x^2+1)^2} = \frac{8-8x^2}{(x^2+1)^2} = \frac{8(1-x^2)}{(x^2+1)^2}$

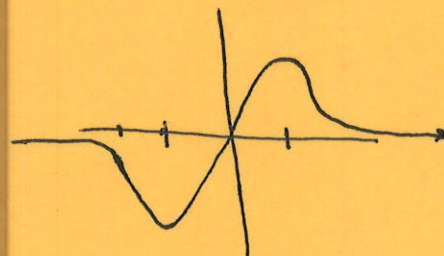
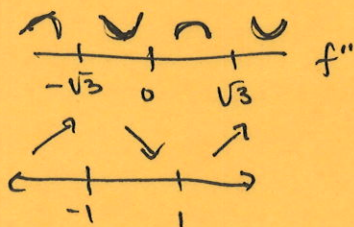
critical points at 1, -1

$8(x^2+1)^{-2}(1-x^2)$

$f''(x) = 8(-2)(x^2+1)^{-3} 2x(1-x^2) + 8(x^2+1)^{-2}(-2x)$

$= \frac{-16(2x)(1-x^2) - 16x(x^2+1)}{(x^2+1)^3} = \frac{-32x + 32x^3 - 16x^3 - 16x}{(x^2+1)^3} = \frac{16x^3 - 48x}{(x^2+1)^3}$

inflection points $16x(x^2-3)$ at $x=0, x = \pm\sqrt{3}$



c. $f(x) = -x\sqrt{1-x^2}$

$f'(x) = -\sqrt{1-x^2} - x(1-x^2)^{-1/2}(\frac{1}{2})(-2x) = -(1-x^2)^{1/2} + x^2(1-x^2)^{-1/2}$

$= \frac{-(1-x^2) + x^2}{(1-x^2)^{1/2}} = \frac{-1+x^2+x^2}{(1-x^2)^{1/2}} = \frac{2x^2-1}{(1-x^2)^{1/2}}$

$x = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$ extrema

$f''(x) = \frac{4x(1-x^2)^{1/2}}{(1-x^2)^{3/2}} - \frac{1}{2}(1-x^2)^{-3/2}(-2x)(2x^2-1) = \frac{4x(1-x^2) + 2x^3-x}{(1-x^2)^{3/2}}$

$\frac{4x - 4x^3 + 2x^3 - x}{(1-x^2)^{3/2}} = \frac{-2x^3 + 3x}{(1-x^2)^{3/2}}$