

Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations.

1. Evaluate each limit.

a. $\lim_{x \rightarrow -1} (x^2 - 4) = (-1)^2 - 4 = 1 - 4 = -3$

b. $\lim_{x \rightarrow 2} \frac{3x^2 + x - 14}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{(3x+7)(x-2)}{(x-2)(x+2)} = \lim_{x \rightarrow 2} \frac{3x+7}{x+2} = \frac{3(2)+7}{2+2} = \frac{13}{4}$

2. For $f(x) = \begin{cases} \frac{1}{3}x + 4, & x < 3 \\ 2x - 1, & x \geq 3 \end{cases}$, evaluate:

a. $\lim_{x \rightarrow 3^-} f(x) = \frac{1}{3}(3) + 4 = 1 + 4 = 5$

b. $\lim_{x \rightarrow 3^+} f(x) = 2(3) - 1 = 6 - 1 = 5$

c. $\lim_{x \rightarrow 3} f(x) = 5$

d. $f(3) = 5$

e. Is the function continuous at 3?

yes, since the limit at 3 exists and is the same as $f(3)$

3. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = 2x + 5$.

$$\lim_{h \rightarrow 0} \frac{2(x+h)+5 - (2x+5)}{h} = \lim_{h \rightarrow 0} \frac{2x+2h+5 - 2x-5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2h}{h} = \lim_{h \rightarrow 0} 2 = 2$$