

Matrices (Overview) Key

(1)

$$i. A+B = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 4 \\ 0 & 4 \end{bmatrix}$$

$$B+C = \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 1 \\ -3 & 4 \end{bmatrix}$$

$$A+C = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -5 & 8 \end{bmatrix}$$

$$D+E = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -3 & 5 \\ 1 & -4 & 0 \\ -1 & 2 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 9 \\ -1 & -3 & 0 \\ 2 & -2 & -6 \end{bmatrix}$$

$$ii. 4A = 4 \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 12 & 4 \\ -4 & 16 \end{bmatrix} \quad -3B = -3 \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -27 & -9 \\ -3 & 0 \end{bmatrix}$$

$$-1D = - \begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -4 \\ 2 & -1 & 0 \\ -3 & 4 & -1 \end{bmatrix}$$

$$2F = 2 \begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & 4 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 6 & -4 & 0 \\ 4 & 8 & -2 & 10 \end{bmatrix}$$

$$21G = 21 \begin{bmatrix} 6 & -7 \\ 11 & -5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 126 & -147 \\ 231 & -105 \\ 42 & 63 \end{bmatrix} \quad -5H = -5 \begin{bmatrix} 1 & 0 & -2 \\ -5 & 0 & 10 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 10 \\ 25 & 0 & -50 \end{bmatrix}$$

$$iii. AB = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 27+1 & 9+0 \\ -9+4 & -3+0 \end{bmatrix} = \begin{bmatrix} 28 & 9 \\ -5 & -3 \end{bmatrix}$$

$$AC = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 6-4 & -6+4 \\ -2-16 & 2+16 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -18 & 18 \end{bmatrix}$$

$$DE = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} \begin{bmatrix} 0 & -3 & 5 \\ 1 & -4 & 0 \\ -1 & 2 & -7 \end{bmatrix} = \begin{bmatrix} 0+3-4 & -3-12+8 & 5+0-28 \\ 0+1+0 & 6-4+0 & -10+0+0 \\ 0-4-1 & -9+16+2 & 15+0-7 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -7 & -23 \\ 1 & 2 & -10 \\ -5 & 9 & 8 \end{bmatrix}$$

iii. can't do

$$BF = \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & 4 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 9+6 & 27+12 & -6-3 & 0+15 \\ 1+0 & 3+0 & -2+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 15 & 39 & -9 & 15 \\ 1 & 3 & -2 & 0 \end{bmatrix}$$

2×2 \quad 2×4
 $\underbrace{\hspace{10em}}$
 ok

$$GC = \begin{bmatrix} 6 & -7 \\ 11 & -5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 12+14 & -12-28 \\ 22+20 & -22-20 \\ 4-12 & -4+12 \end{bmatrix} = \begin{bmatrix} 26 & -40 \\ 42 & -42 \\ -8 & 8 \end{bmatrix}$$

3×2 \quad 2×2
 $\underbrace{\hspace{10em}}$
 ok

$$DG = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} \begin{bmatrix} 6 & -7 \\ 11 & -5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 6+33+8 & -7-15+12 \\ -12+11+0 & 14-5+0 \\ 18-44+2 & -21+20+3 \end{bmatrix} = \begin{bmatrix} 47 & -10 \\ -1 & 9 \\ -24 & 2 \end{bmatrix}$$

3×3 \quad 3×2
 $\underbrace{\hspace{10em}}$
 ok

$$EH = \begin{bmatrix} 0 & -3 & 5 \\ 1 & -4 & 0 \\ -1 & 2 & -7 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \end{bmatrix}$$

3×3 \quad 1×3
 $\underbrace{\hspace{10em}}$
 ok

not defined
mismatched dimensions

$$BJ = \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 36-3 \\ 4+0 \end{bmatrix} = \begin{bmatrix} 33 \\ 4 \end{bmatrix}$$

2×2 \quad 2×1
 $\underbrace{\hspace{10em}}$
 ok

iv. $A^T = \begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix}$ $D^T = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 1 & -4 \\ 4 & 0 & 1 \end{bmatrix}$ $F^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -2 & -1 \\ 0 & 5 \end{bmatrix}$ $G^T = \begin{bmatrix} 6 & 11 & 2 \\ -7 & -5 & 3 \end{bmatrix}$

$J^T = \begin{bmatrix} 4 & -1 \end{bmatrix}$

v. $A^{-1} = \frac{1}{12+1} \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4/13 & -1/13 \\ 1/13 & 3/13 \end{bmatrix}$ $B^{-1} = \frac{1}{0-3} \begin{bmatrix} 0 & -3 \\ -1 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1/3 & -3 \end{bmatrix}$

$$C^{-1} = \frac{1}{8-8} \begin{bmatrix} 4 & 2 \\ 4 & 2 \end{bmatrix} \Rightarrow \text{does not exist} \quad \det C = 0$$

$$D^{-1} = \begin{bmatrix} 1/27 & -19/27 & -4/27 \\ 2/27 & -11/27 & -8/27 \\ 5/27 & 13/27 & 7/27 \end{bmatrix} \quad E^{-1} = \begin{bmatrix} -28/31 & 11/31 & -20/31 \\ -7/31 & -5/31 & -5/31 \\ 2/31 & -3/31 & -1/31 \end{bmatrix}$$

ix d.

(4)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ -1 & 2 & -3 & 14 \\ 3 & -5 & -2 & -18 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 123/31 \\ 0 & 1 & 0 & 205/31 \\ 0 & 0 & 1 & -49/31 \end{array} \right]$$

or $\left[\begin{array}{ccc|c} 1 & -9/3 & -2/3 & -6 \\ 0 & 1 & 5/8 & 45/8 \\ 0 & 0 & 1 & -49/31 \end{array} \right]$ and backsolve to get
 $x = 123/31, y = 205/31, z = -49/31$

x. a. $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1/14 & 3/14 \\ 2/7 & -1/7 \end{bmatrix}$ $\begin{bmatrix} 1/14 & 3/14 \\ 2/7 & -1/7 \end{bmatrix} \begin{bmatrix} 12 \\ 10 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $x=3, y=2$

b. $\begin{bmatrix} -1 & 5 \\ 3 & -4 \end{bmatrix}^{-1} = \begin{bmatrix} 4/11 & 5/11 \\ 3/11 & 1/11 \end{bmatrix}$ $\begin{bmatrix} 4/11 & 5/11 \\ 3/11 & 1/11 \end{bmatrix} \begin{bmatrix} 17 \\ 12 \end{bmatrix} = \begin{bmatrix} 128/11 \\ 63/11 \end{bmatrix}$ $x = 128/11, y = 63/11$

c. $\begin{bmatrix} 5 & -1 & 2 \\ 3 & 2 & -4 \\ -4 & -3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 2/13 & 1/13 & 0 \\ -1/5 & -1/5 & -2/5 \\ 1/65 & -19/65 & -1/5 \end{bmatrix}$

$$\begin{bmatrix} 2/13 & 1/13 & 0 \\ -1/5 & -1/5 & -2/5 \\ 1/65 & -19/65 & -1/5 \end{bmatrix} \begin{bmatrix} 10 \\ 16 \\ 7 \end{bmatrix} = \begin{bmatrix} 36/13 \\ -8 \\ -77/13 \end{bmatrix}$$

$x = 36/13$ $y = -8$
 $z = -77/13$

d. $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & -3 \\ 3 & -5 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} 19/31 & 3/31 & 5/31 \\ 11/31 & 5/31 & -2/31 \\ 1/31 & -8/31 & -3/31 \end{bmatrix}$

$$\begin{bmatrix} 19/31 & 3/31 & 5/31 \\ 11/31 & 5/31 & -2/31 \\ 1/31 & -8/31 & -3/31 \end{bmatrix} \begin{bmatrix} 9 \\ 14 \\ -18 \end{bmatrix} = \begin{bmatrix} 123/31 \\ 205/31 \\ -49/31 \end{bmatrix}$$

$x = 123/31$ $y = 205/31$
 $z = -49/31$

xi. a. $\det \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} = -14$ $\det \begin{pmatrix} 12 & 3 \\ 10 & -1 \end{pmatrix} = -42$ $\det \begin{pmatrix} 2 & 12 \\ 4 & 10 \end{pmatrix} = -28$

$$x = \frac{-42}{-14} = 3 \quad y = \frac{-28}{-14} = 2$$

b. $\det \begin{pmatrix} -1 & 5 \\ 3 & -4 \end{pmatrix} = -11$ $\det \begin{pmatrix} 17 & 5 \\ 12 & -4 \end{pmatrix} = -128$ $\det \begin{pmatrix} -1 & 17 \\ 3 & 12 \end{pmatrix} = -63$

$$x = \frac{-128}{-11} = \frac{128}{11} \quad y = \frac{-63}{-11} = \frac{63}{11}$$

xi (contd.)

(5)

$$c. \det \begin{pmatrix} 5 & -1 & 2 \\ 3 & 2 & -4 \\ -4 & -3 & 1 \end{pmatrix} = -65$$

$$\det \begin{pmatrix} 10 & -1 & 2 \\ 16 & 2 & -4 \\ 7 & -3 & 1 \end{pmatrix} = -180$$

$$\det \begin{pmatrix} 5 & 10 & 2 \\ 3 & 16 & -4 \\ -4 & 7 & 1 \end{pmatrix} = 520$$

$$\det \begin{pmatrix} 5 & -1 & 10 \\ 3 & 2 & 16 \\ -4 & -3 & 7 \end{pmatrix} = 385$$

$$x = \frac{-180}{-65} = \frac{36}{13}$$

$$y = \frac{520}{-65} = -8$$

$$z = \frac{385}{-65} = -\frac{77}{13}$$

$$d. \det \begin{pmatrix} 1 & 1 & 1 \\ -1 & 2 & -3 \\ 3 & -5 & -2 \end{pmatrix} = -31$$

$$\det \begin{pmatrix} 9 & 1 & 17 \\ 14 & 2 & -3 \\ -18 & -5 & -2 \end{pmatrix} = -123$$

$$\det \begin{pmatrix} 1 & 9 & 1 \\ -1 & 14 & -3 \\ 3 & -18 & -2 \end{pmatrix} = -205$$

$$\det \begin{pmatrix} 1 & 1 & 9 \\ -1 & 2 & 14 \\ 3 & -5 & -18 \end{pmatrix} = 49$$

$$x = \frac{-123}{-31} = \frac{123}{31}$$

$$y = \frac{-205}{-31} = \frac{205}{31}$$

$$z = \frac{49}{-31} = -\frac{49}{31}$$