202 Homework #10 key

1. a. $P = \begin{bmatrix} .7 & .1 & .05 \\ .2 & .5 & .2 \\ .1 & .4 & .75 \end{bmatrix}$

b. $\vec{x}_0 = \begin{bmatrix} .8 \\ .1 \\ .1 \end{bmatrix}$ $\vec{x}_1 = P\vec{x}_0 = \begin{bmatrix} .575 \\ .23 \\ .195 \end{bmatrix}$

c. $P^{\wedge}\text{big}(120) = \begin{bmatrix} 9/49 & 9/49 & 9/49 \\ 2/7 & 2/7 & 2/7 \\ 26/49 & 26/49 & 26/49 \end{bmatrix}$ $\vec{x}_\infty = \begin{bmatrix} 9/49 \\ 2/7 \\ 26/49 \end{bmatrix}$

2. $\begin{bmatrix} .75 & .70 & .10 & 0 \\ .22 & .25 & .40 & 0 \\ .027 & .045 & .45 & 0 \\ .003 & .005 & .05 & 1 \end{bmatrix}$ $\vec{x}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$P\vec{x}_0 = \vec{x}_1 = \begin{bmatrix} .75 \\ .22 \\ .027 \\ .003 \end{bmatrix}$ in people 750 healthy
220 ill
27 very ill
3 dead

around 500 steps or roughly 41 years (since each step is a month)

3. a. There is one equilibrium vector since there is communication between all states

$P - I = \begin{bmatrix} -.3 & .4 \\ .3 & -.4 \end{bmatrix}$ $.3x_1 = .4x_2$, $x_1 = \frac{4}{3}x_2$, $x_2 = x_2$ $\vec{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $4+3=7$ $\vec{q} = \begin{bmatrix} 4/7 \\ 3/7 \end{bmatrix}$

b. communication between all states; one $\vec{q}$ $11+15+21=47$

$P - I = \begin{bmatrix} -.6 & .3 & .1 \\ .3 & -.5 & .2 \\ .3 & .2 & -.3 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -11/21 \\ 0 & 1 & -5/7 \\ 0 & 0 & 0 \end{bmatrix}$ $x_1 = 11/21\,x_3$, $x_2 = 5/7\,x_3$, $x_3 = x_3$ $\vec{v} = \begin{bmatrix} 11 \\ 15 \\ 21 \end{bmatrix}$ $\vec{q} = \begin{bmatrix} 11/47 \\ 15/47 \\ 21/47 \end{bmatrix}$

c. there is communication, but x_2 is an absorbing state so $q = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

d. there are two q's since $A \leftrightarrow D$ and $B \leftrightarrow C$ only. (basis dim = 2)

$\begin{bmatrix} -.3 & 0 & 0 & .4 \\ 0 & -.5 & .2 & 0 \\ 0 & .5 & -.2 & 0 \\ .3 & 0 & 0 & -.4 \end{bmatrix} \xrightarrow{\text{rref}}$ $.3x_1 = .4x_4$, $x_4 = x_4$, $x_2 = 0$, $x_3 = 0$, $x_1 = \frac{4}{3}x_4$ $\vec{v}_1 = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 3 \end{bmatrix} \Rightarrow \vec{q}_1 = \begin{bmatrix} 4/7 \\ 0 \\ 0 \\ 3/7 \end{bmatrix}$ $4+3=7$

$.5x_2 = .2x_3$, $x_2 = \frac{2}{5}x_3$, $x_3 = x_3$, $x_1 = 0$, $x_4 = 0$ $\vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 5 \\ 0 \end{bmatrix}$ $\vec{q}_2 = \begin{bmatrix} 0 \\ 2/7 \\ 5/7 \\ 0 \end{bmatrix}$ $2+5=7$