

202 Homework #1 key

①

a. $\left[\begin{array}{cc|c} 3 & 6 & -3 \\ 5 & 7 & 10 \end{array} \right] \quad \frac{1}{3}R_1 \rightarrow R_1 \quad \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 5 & 7 & 10 \end{array} \right] \quad \begin{array}{l} -5R_1 + R_2 \rightarrow R_2 \\ -5 \quad -10 \quad 5 \end{array} \quad \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & -3 & 15 \end{array} \right]$

$-\frac{1}{3}R_2 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 1 & -5 \end{array} \right] \quad -2R_2 + R_1 \rightarrow R_1 \quad \left[\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & -5 \end{array} \right] \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

b. $\left[\begin{array}{ccc|c} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{array} \right] \quad \frac{1}{2}R_1 \rightarrow R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{array} \right] \quad \begin{array}{l} -3R_1 + R_3 \rightarrow R_3 \\ -3 \quad 0 \quad 9 \quad 12 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & 7 & 8 \end{array} \right] \quad -6R_2 + R_3 \rightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 5 & -10 \end{array} \right] \quad \frac{1}{5}R_3 \rightarrow R_3$

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & -2 \end{array} \right] \quad \begin{array}{l} x_3 = -2 \\ x_2 + 2x_3 = 3 \\ x_2 + 2(-2) = 3 \end{array} \rightarrow \begin{array}{l} x_2 - 4 = 3 \quad x_2 = 7 \\ x_1 - 3x_3 = -4 \\ x_1 - 3(-2) = -4 \end{array} \rightarrow \begin{array}{l} x_1 + 6 = -4 \\ x_1 = -10 \end{array}$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 7 \\ -2 \end{bmatrix}$

c. $\left[\begin{array}{cccc|c} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{array} \right] \quad \frac{1}{2}R_1 \rightarrow R_1 \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{array} \right] \quad \begin{array}{l} 3R_1 + R_4 \rightarrow R_4 \\ 3 \quad 0 \quad 0 \quad 6 \quad -15 \end{array}$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 2 & 3 & 7 & -10 \end{array} \right] \quad \frac{1}{3}R_2 \rightarrow R_2 \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 2 & 3 & 7 & -10 \end{array} \right] \quad -2R_2 + R_4 \rightarrow R_4$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 1 & 7 & -10 \end{array} \right] \quad -R_3 + R_4 \rightarrow R_4 \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & 3 & -9 \end{array} \right] \quad \frac{1}{3}R_4 \rightarrow R_4$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right] \quad \begin{array}{l} x_4 = -3 \\ x_3 + 4(-3) = -1 \\ x_3 - 12 = -1 \end{array} \rightarrow \begin{array}{l} x_3 = 11 \\ x_2 = -11 \\ x_1 - 2(-3) = -5 \end{array} \rightarrow \begin{array}{l} x_1 + 6 = -5 \\ x_1 = -11 \end{array}$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -11 \\ -11 \\ 11 \\ -3 \end{bmatrix}$

d. $\left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ -3 & -4 & 2 & 0 \end{array} \right] \quad \frac{1}{2}R_1 \rightarrow R_1 \quad \left[\begin{array}{ccc|c} 1 & -\frac{5}{2} & 4 & 0 \\ -3 & -4 & 2 & 0 \end{array} \right] \quad \begin{array}{l} 3R_1 + R_2 \rightarrow R_2 \\ 3 \quad -\frac{15}{2} \quad 12 \quad 0 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -\frac{5}{2} & 4 & 0 \\ 0 & -\frac{23}{2} & 14 & 0 \end{array} \right]$