

202 Homework #3 key

①

1a.  $\det \begin{pmatrix} -1 & 3 \\ 4 & 2 \end{pmatrix} = (-1)(2) - 4(3) = -2 - 12 = -14$

b.  $\begin{vmatrix} 4 & 3 & 0 \\ 6 & 5 & 2 \\ 9 & 7 & 3 \end{vmatrix} = 4 \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} - 3 \begin{vmatrix} 6 & 2 \\ 9 & 3 \end{vmatrix} + 0 \begin{vmatrix} 6 & 5 \\ 9 & 7 \end{vmatrix} =$   
 $4(15 - 14) - 3(18 - 18) = 4(1) = 4$

c.  $\begin{vmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix} = 2 \begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 8 \end{vmatrix} = 2(5) \begin{vmatrix} 7 & 2 \\ 3 & 1 \end{vmatrix} = 10(7 - 6) = 10$

d.  $\begin{vmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{vmatrix} = 1 \begin{vmatrix} 6 & 3 & 2 & 4 \\ 9 & 0 & -4 & 1 \\ 3 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 \end{vmatrix} = 3 \begin{vmatrix} 3 & 2 & 4 \\ 0 & -4 & 1 \\ 2 & 3 & 2 \end{vmatrix} = 3 \left[ 3 \begin{vmatrix} -4 & 1 \\ 3 & 2 \end{vmatrix} + \right.$   
 $\left. -0 \begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} + 2 \begin{vmatrix} 2 & 4 \\ -4 & 1 \end{vmatrix} \right] = 3 \left[ 3(-8 - 3) + 2(2 + 16) \right] =$   
 $3[-33 + 36] = 3(3) = 9$

e.  $\begin{vmatrix} k & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = k(1)(1) = k$

f.  $\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$

2a.  $\begin{bmatrix} 1 & 5 & -3 \\ 3 & -3 & 3 \\ 2 & 13 & -7 \end{bmatrix} \begin{array}{l} -3R_1 + R_2 \rightarrow R_2 \\ -2R_1 + R_3 \rightarrow R_3 \\ \text{no change} \end{array} \begin{bmatrix} 1 & 5 & -3 \\ 0 & -18 & 12 \\ 0 & 3 & -1 \end{bmatrix} \begin{array}{l} R_2 \leftrightarrow R_3 \\ (-1) \end{array} \begin{bmatrix} 1 & 5 & -3 \\ 0 & 3 & -1 \\ 0 & -18 & 12 \end{bmatrix}$   
 $6R_2 + R_3 \rightarrow R_3 \begin{bmatrix} 1 & 5 & -3 \\ 0 & 3 & -1 \\ 0 & 0 & 6 \end{bmatrix} \det(A) = (-1)(1)(3)(6) = -18$

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(2)

2b. 
$$\begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & -2 & 3 \end{bmatrix}$$
  $R_1 + R_3 \rightarrow R_3$   
 $-3R_1 + R_4 \rightarrow R_4$   
 no change 
$$\begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 1 & 5 & 5 \\ 0 & 2 & 7 & 3 \end{bmatrix}$$

$-R_2 + R_3 \rightarrow R_3$   
 $-2R_2 + R_4 \rightarrow R_4$   
 no change 
$$\begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -3 & -5 \end{bmatrix}$$
  $R_3 \leftrightarrow R_4$   
 $(-1)$  
$$\begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & -3 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\det(A) = (-1)(1)(1)(-3)(1) = 3$

2c. 
$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{bmatrix}$$
  $2R_1 + R_3 \rightarrow R_3$   
 $-3R_1 + R_4 \rightarrow R_4$   
 $-3R_1 + R_5 \rightarrow R_5$   
 no change 
$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & -3 & 1 & 3 & 7 \\ 0 & -2 & 0 & 8 & -1 \\ 0 & -4 & 8 & 2 & 13 \end{bmatrix}$$

$\frac{3}{2}R_2 + R_3 \rightarrow R_3$   
 $R_2 + R_4 \rightarrow R_4$   
 $2R_2 + R_5 \rightarrow R_5$   
 no change 
$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & -5 & \frac{3}{2} & -2 \\ 0 & 0 & -4 & 7 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
  $-\frac{4}{5}R_3 + R_4 \rightarrow R_4$   
 $0 \ 0 \ 4 \ -\frac{6}{5} \ \frac{8}{5}$   
 no change

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & -5 & \frac{3}{2} & -2 \\ 0 & 0 & 0 & \frac{29}{5} & -\frac{27}{5} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
  $\det(A) = 1(2)(-5)(\frac{29}{5})(1) = -58$

d. 
$$\begin{bmatrix} -3 & -2 & 1 & -4 \\ 1 & 3 & 0 & -3 \\ -3 & 4 & -2 & 8 \\ 3 & -4 & 0 & 4 \end{bmatrix}$$
  $R_1 + R_4 \rightarrow R_4$   
 $-R_1 + R_3 \rightarrow R_3$   
 no change 
$$\begin{bmatrix} -3 & -2 & 1 & -4 \\ 1 & 3 & 0 & -3 \\ 0 & 6 & -3 & 12 \\ 0 & -6 & 1 & 0 \end{bmatrix}$$
  $R_1 \leftrightarrow R_2$   
 $(-1)$

$$\begin{bmatrix} 1 & 3 & 0 & -3 \\ -3 & -2 & 1 & -4 \\ 0 & 6 & -3 & 12 \\ 0 & -6 & 1 & 0 \end{bmatrix}$$
  $3R_1 + R_2 \rightarrow R_2$   
 no change 
$$\begin{bmatrix} 1 & 3 & 0 & -3 \\ 0 & 7 & 1 & -13 \\ 0 & 6 & -3 & 12 \\ 0 & -6 & 1 & 0 \end{bmatrix}$$
  $R_2 + R_4 \rightarrow R_4$