202 Homework #5 key

1. a. true

b. false. $A\vec{x}=\vec{b}$ is a matrix equation. $\vec{a}_1x_1+\vec{a}_2x_2+\vec{a}_3x_3+\dots+\vec{a}_nx_n=\vec{b}$ is a vector equation.

c. false. pivot in every column (but the last), or rows of zeros are consistent. If the augmented matrix has a pivot in last row last column it is inconsistent.

d. false. There may be $\vec{b}$ in $\mathbb{R}^m$ (if $m>n$) where the system is inconsistent.

e. false. trivial solution has no free variables.

f. false. line through $\vec{p}$, parallel to $\vec{v}$.

g. true

h. true. as long as the vector is not $\vec{0}$

i. false. the set also needs to be independent.

j. true

k. false. as small as possible

l. false.

m. false. must also span H.

n. true.

o. true

p. false. its in $\mathbb{R}^n$

q. true

r. true

s. false. Set of all $\vec{b}$'s so that $A\vec{x}=\vec{b}$ has a solution.

t. false

u. false. pivot columns must be taken from A.

v. true

w. false. P_3 is isomorphic to $\mathbb{R}^4$.

x. true

y. true

z. true

aa. false. The # of columns of A.

bb. true

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1cc. true

dd. true

ee. true

ff. false. only if the matrix is $n \times n$.

gg. false. system could be dependent.

hh. false. $P_B[\vec{x}]_B = \vec{x}$

ii. false. They are C-coordinate vectors of the vectors in B.

jj. true

2\. a. $\begin{bmatrix} 1 & * \\ 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

c. 4 pivots, one for each column. It can't have more than that.

3\. $\begin{bmatrix} 2 \\ 6 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} -8 \\ 0 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -6 \\ 6 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 10 \\ 6 \\ -6 \\ 2 \end{bmatrix}$ etc. answers will vary

4\. a. rref $\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ they do not span $\mathbb{R}^4$, only 3 pivots.

b. rref $\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ they do span $\mathbb{R}^4$ as there are 4 pivots.

5\. a. independent; one pivot in each column

b. reduces to $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ There is a pivot in every column, so independent

c. 2 vectors, not multiples, so independent.

d. 4 vectors, more than rows of matrix, dependent.

6\. a. no pivot in column 3 - remove vector; basis is $\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 10 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -6 \\ 9 \end{bmatrix} \right\}$

b. no pivot in column 4 - remove; basis is $\left\{ \begin{bmatrix} -3 \\ 2 \\ 6 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -9 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -4 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -6 \\ 3 \\ 0 \\ -1 \\ 0 \end{bmatrix} \right\}$