

202 Homework #6 Key

①

1. a. $\vec{u} \cdot \vec{v}$ i. $(-1)(4) + 2(6) = -4 + 12 = 8$

ii. $(12)(2) + 3(-3) + (-5)(3) = 24 - 9 - 15 = 0$

iii. $(3)(-4) + 2(1) + (-5)(-2) + 0(6) = -12 + 2 + 10 + 0 = 0$

b. $\|\vec{u}\|$ and $\|\vec{v}\|$

i. $\|\vec{u}\| = \sqrt{1^2 + 2^2} = \sqrt{5}$ $\|\vec{v}\| = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$

ii. $\|\vec{u}\| = \sqrt{144 + 9 + 25} = \sqrt{178}$ $\|\vec{v}\| = \sqrt{4 + 9 + 9} = \sqrt{22}$

iii. $\|\vec{u}\| = \sqrt{9 + 4 + 25} = \sqrt{38}$ $\|\vec{v}\| = \sqrt{16 + 1 + 4 + 36} = \sqrt{57}$

c. $\frac{\vec{u}}{\|\vec{u}\|}$ and $\frac{\vec{v}}{\|\vec{v}\|}$

i. $\begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix} = \hat{u}$, $\hat{v} = \begin{bmatrix} 2/\sqrt{13} \\ 3/\sqrt{13} \end{bmatrix}$

iii. $\hat{u} = \begin{bmatrix} 3/\sqrt{38} \\ 2/\sqrt{38} \\ -5/\sqrt{38} \\ 0 \end{bmatrix}$, $\hat{v} = \begin{bmatrix} -4/\sqrt{57} \\ 1/\sqrt{57} \\ -2/\sqrt{57} \\ 6/\sqrt{57} \end{bmatrix}$

ii. $\hat{u} = \begin{bmatrix} 12/\sqrt{178} \\ 3/\sqrt{178} \\ -5/\sqrt{178} \end{bmatrix}$, $\hat{v} = \begin{bmatrix} 2/\sqrt{22} \\ -3/\sqrt{22} \\ 3/\sqrt{22} \end{bmatrix}$

d. $\|\vec{u}\|^2 + \|\vec{v}\|^2$ i. $5 + 52 = 57$

ii. $178 + 22 = 200$

iii. $38 + 57 = 95$

e. $\|\vec{u} + \vec{v}\|^2$

i. $\vec{u} + \vec{v} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$ $\|\vec{u} + \vec{v}\| = \sqrt{9 + 64} = \sqrt{73}$ $\|\vec{u} + \vec{v}\|^2 = 73$

ii. $\vec{u} + \vec{v} = \begin{bmatrix} 14 \\ 0 \\ -2 \end{bmatrix}$ $\|\vec{u} + \vec{v}\| = \sqrt{196 + 4} = \sqrt{200}$ $\|\vec{u} + \vec{v}\|^2 = 200$

iii. $\vec{u} + \vec{v} = \begin{bmatrix} -1 \\ 3 \\ -7 \\ 6 \end{bmatrix}$ $\|\vec{u} + \vec{v}\| = \sqrt{1 + 9 + 49 + 36} = \sqrt{95}$ $\|\vec{u} + \vec{v}\|^2 = 95$

f. $\left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|}\right) \vec{v}$

i. $\frac{8}{52} \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \frac{2}{13} \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 8/13 \\ 12/13 \end{bmatrix}$

iii. $\frac{0}{57} \begin{bmatrix} -4 \\ 1 \\ -2 \\ 6 \end{bmatrix} = \vec{0}$

ii. $\frac{0}{22} \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix} = \vec{0}$

1g. $\|\vec{u} - \vec{v}\| =$

i. $\vec{u} - \vec{v} = \begin{bmatrix} -5 \\ -4 \end{bmatrix}$ $\|\vec{u} - \vec{v}\| = \sqrt{25+16} = \sqrt{41}$

ii. $\vec{u} - \vec{v} = \begin{bmatrix} 10 \\ 6 \\ -8 \end{bmatrix}$ $\|\vec{u} - \vec{v}\| = \sqrt{100+36+64} = \sqrt{200}$

iii. $\vec{u} - \vec{v} = \begin{bmatrix} 7 \\ 1 \\ -3 \\ -6 \end{bmatrix}$ $\|\vec{u} - \vec{v}\| = \sqrt{49+1+9+36} = \sqrt{95}$

2a. See 1g.

b. ii & iii are. (see 1a)

c. for ii & iii, $\pi/2$; for i. $\cos \theta = \frac{8}{\sqrt{5} \cdot 2\sqrt{13}} = \frac{4}{\sqrt{65}}$, $\theta = \cos^{-1}\left(\frac{4}{\sqrt{65}}\right) = 1.05165$ radians
 $\approx 60.255^\circ$

d. See 1c.

e. See 1f.

f. for ii & iii, use the unit vectors in 1f.

g. ii. $12a + 3b - 5c = 0$
 $2a - 3b + 3c = 0$

rref $\Rightarrow \begin{bmatrix} 1 & 0 & -1/7 \\ 0 & 1 & -23/21 \end{bmatrix}$

$w_3 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$w^\perp = \left\{ \begin{bmatrix} 3 \\ 23 \\ 21 \end{bmatrix} \right\}$

$x_1 = 1/7 x_3$

$x_2 = 23/21 x_3$

$x_3 = x_3$

$\Rightarrow \vec{w}_3 = \begin{bmatrix} 3 \\ 23 \\ 21 \end{bmatrix}$ $\sqrt{9+529+441} = \sqrt{979}$

$\vec{w}_3 = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

iii. $3a + 2b - 5c + 0d = 0$
 $-4a + b - 2c + 6d = 0$

rref $\Rightarrow \begin{bmatrix} 1 & 0 & -1/11 & -12/11 \\ 0 & 1 & -24/11 & 18/11 \end{bmatrix}$

$x_1 = 1/11 x_3 + 12/11 x_4$

$x_2 = 26/11 x_3 - 18/11 x_4$

$x_3 = x_3$

$x_4 = x_4$

$\begin{bmatrix} 11 \\ 26 \\ 11 \\ 0 \end{bmatrix}$

$w_4 = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$3a + 2b - 5c + 0d = 0$

$-4a + b - 2c + 6d = 0$

$a + 26b + 11c + 0d = 0$

rref $\Rightarrow \begin{bmatrix} 1 & 0 & 0 & -8/7 \\ 0 & 1 & 0 & 2/7 \\ 0 & 0 & 1 & -4/7 \end{bmatrix}$

$x_1 = 8/7 x_4$

$x_2 = -2/7 x_4$

$x_3 = 4/7 x_4$

$x_4 = x_4$

$\begin{bmatrix} 8 \\ -2 \\ 4 \\ 7 \end{bmatrix}$

$w^\perp = \left\{ \begin{bmatrix} 1 \\ 26 \\ 11 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ -2 \\ 4 \\ 7 \end{bmatrix} \right\}$