

1a. $A = \begin{bmatrix} 3 & -5 \\ 3 & 2 \\ 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Counterclockwise

b. $A = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

2a. $p(t) = at^2 + bt + c$ $(t+3)(at^2 + bt + c) = at^3 + bt^2 + ct + 3at^2 + 3bt + 3c$

c. $\begin{bmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

b. It is linear since

i) $(t+3)p(t) = \vec{0}$ when $p(t) = \vec{0}$

ii) $(t+3)p(t) + q(t)(t+3) = (t+3)(p(t) + q(t))$

iii) $k(t+3)p(t) = (t+3)[kp(t)]$.

and it's written as a matrix, so it's linear.

3a. $\begin{bmatrix} 2 & -4 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 2 \\ 1 & 4 \\ 0 & 1 \end{bmatrix}$

c. $(2t^2 - t + 6)(a_0 + a_1t + a_2t^2 + a_3t^3) = 6a_0 + 6a_1t + 6a_2t^2 + 6a_3t^3 - a_0t - a_1t^2 - a_2t^3 - a_3t^4 + 2a_0t^2 + 2a_1t^3 + 2a_2t^4 + 2a_3t^5$

$$\begin{bmatrix} 6 & 0 & 0 & 0 \\ -1 & 6 & 0 & 0 \\ 2 & -1 & 6 & 0 \\ 0 & 2 & -1 & 6 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$2d. p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \Rightarrow$$

$$p'(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$2e. a_1 e^x + a_2 e^{-x} + a_3 e^{5x} + a_4 e^{-7x} = y(x)$$

$$y'(x) = a_1 e^x + (-1)a_2 e^{-x} + 5a_3 e^{5x} - 7a_4 e^{-7x}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

$$f. y(x) = a_1 e^{3x} \cos(2x) + a_2 e^{3x} \sin(2x)$$

$$y'(x) = a_1 3e^{3x} \cos 2x + (-2)a_1 e^{3x} \sin 2x + 3a_2 e^{3x} \sin 2x + 2a_2 e^{3x} \cos 2x$$

$$\begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix}$$

$$g. 225^\circ = 5\pi/4$$

$$\begin{bmatrix} \cos(5\pi/4) & -\sin(5\pi/4) \\ \sin(5\pi/4) & \cos(5\pi/4) \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

h.

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & -1/2 \end{bmatrix} = \begin{bmatrix} 0 & -\sqrt{3}/2 & -1/2 \\ 0 & -1 & \sqrt{3} \\ 4 & 0 & 0 \end{bmatrix}$$

$$4a. P_c^{-1} P_B = \begin{bmatrix} 13/9 & -14/9 \\ -2/9 & -2/9 \end{bmatrix}$$

$$c. P_c^{-1} P_B = \begin{bmatrix} -11/17 & 8/17 & 22/17 & 25/17 \\ -12/17 & 1/17 & -10/17 & -50/17 \\ 22/17 & 35/17 & 7/17 & 1/17 \\ 7/17 & -2/17 & -14/17 & -2/17 \end{bmatrix}$$

$$b. P_c^{-1} P_B = \begin{bmatrix} 5/8 & 1/2 & -5/8 \\ 17/8 & -1/2 & -17/8 \\ 1/8 & 1/2 & 7/8 \end{bmatrix}$$