

202 homework #9 key

①

1a.  $A - \lambda I = \begin{bmatrix} 1-\lambda & -2 \\ 1 & 3-\lambda \end{bmatrix} \Rightarrow (1-\lambda)(3-\lambda) + 2 = \lambda^2 - 4\lambda + 3 + 2 = \lambda^2 - 4\lambda + 5 = 0$   
 $\lambda = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$

$\lambda_1 = 2+i$

$\begin{bmatrix} 1-2-i & -2 \\ 1 & 3-2-i \end{bmatrix} = \begin{bmatrix} -1-i & -2 \\ 1 & 1-i \end{bmatrix}$   $x_1 + (1-i)x_2 = 0$   
 $x_1 = -(1-i)x_2$   $\vec{v}_1 = \begin{bmatrix} 1-i \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} i$   
 $x_2 = x_2$

$\lambda_2 = 2-i$   $\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i$

$\lambda = a - bi$

$P = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$   $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$   $PCP^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

b.  $\begin{bmatrix} 0-\lambda & 5 \\ -2 & 2-\lambda \end{bmatrix} \Rightarrow (-\lambda)(2-\lambda) + 10 = 0$   $\lambda = \frac{2 \pm \sqrt{4-40}}{2} = \frac{2 \pm 6i}{2}$   
 $\lambda^2 - 2\lambda + 10 = 0$   $= 1 \pm 3i$

$\lambda_1 = 1+3i$

$\begin{bmatrix} -1-3i & 5 \\ -2 & 2-1-3i \end{bmatrix} = \begin{bmatrix} -1-3i & 5 \\ -2 & 1-3i \end{bmatrix}$   $-2x_1 + (1-3i)x_2 = 0$   
 $x_1 = \frac{1-3i}{2} x_2$   $\vec{v}_1 = \begin{bmatrix} 1-3i \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \end{bmatrix} i$   
 $x_2 = x_2$

$P = \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix}$   $C = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$

$\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \end{bmatrix} i$

c.  $\begin{bmatrix} -3-\lambda & -8 \\ 4 & 5-\lambda \end{bmatrix} \Rightarrow (-3-\lambda)(5-\lambda) + 32 = \lambda^2 - 2\lambda - 15 + 32 = \lambda^2 - 2\lambda + 17 = 0$   
 $\lambda = \frac{2 \pm \sqrt{4-68}}{2} = \frac{2 \pm 8i}{2} = 1 \pm 4i$

$\lambda_1 = 1+4i$

$\begin{bmatrix} -3-1-4i & -8 \\ 4 & 5-1-4i \end{bmatrix} = \begin{bmatrix} -4-4i & -8 \\ 4 & 4-4i \end{bmatrix} \div 4$   $x_1 + (1-i)x_2 = 0$   
 $x_1 = -(1-i)x_2$   $\vec{v}_1 = \begin{bmatrix} 1-i \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} i$   
 $x_2 = x_2$

$P = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$   $C = \begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix}$

d.  $\begin{bmatrix} -3-\lambda & 7 \\ 5 & -1-\lambda \end{bmatrix} \Rightarrow (-3-\lambda)(-1-\lambda) - 35 = \lambda^2 + 4\lambda + 3 - 35 = \lambda^2 + 4\lambda - 32 = 0$   
 $(\lambda+8)(\lambda-4) = 0$   $\lambda = -8, 4$