202 Proof Set #1 key

1. $1^3+2^3+3^3+\cdots n^3 = \sum_{i=1} i^3 = \dfrac{n^2(n+1)^2}{4}$

a) base case $\sum_{i=1}^{1} 1^3 = \dfrac{1^2(1+1)^2}{4} = 1$ works

b) next case

Suppose formula works for n.

$$1^3+2^3+\cdots+n^3+(n+1)^3 = \sum_{i=1}^{n+1} i^3 = \sum_{i=1}^{n} i^3 + (n+1)^3 = \frac{n^2(n+1)^2}{4} + (n+1)^3$$

$$= \frac{(n+1)^2}{4}\left[n^2+4(n+1)\right] = \frac{(n+1)^2}{4}\left[n^2+4n+4\right] = \frac{(n+1)^2(n+2)^2}{4}$$

which is what we expect for the formula substituting $n+1$ for n.

therefore, by mathematical induction, $1^3+2^3+3^3+\cdots+n^3 = \dfrac{n^2(n+1)^2}{4}$. //

2. $(a+b)^3 = a^3+b^3$? if $a \neq 0$, $b \neq 0$

Suppose $a=2$, $b=3$

$(a+b)^3 = (2+3)^3 = 5^3 = 125$, but

$2^3+3^3 = 8+27 = 35$. $125 \neq 35$ Therefore, the expression

$(a+b)^3 = a^3+b^3$ is false. //

3. $\left[\begin{array}{cc|c} a & b & e \\ c & d & f \end{array}\right]$ is our augmented matrix. To obtain a condition on a unique solution we need to put the matrix in echelon form.

$$\left[\begin{array}{cc|c} a & b & e \\ c & d & f \end{array}\right] \xrightarrow{\frac{1}{a}R_1 \to R_1} \left[\begin{array}{cc|c} 1 & b/a & e/a \\ c & d & f \end{array}\right] \xrightarrow{-cR_1+R_2 \to R_2} \left[\begin{array}{cc|c} 1 & b/a & e/a \\ 0 & d-cb/a & f-ce/a \end{array}\right]$$

$$\xrightarrow{aR_2 \to R_2} \left[\begin{array}{cc|c} 1 & b/a & e/a \\ 0 & ad-bc & fa-ce \end{array}\right]$$

for this matrix to have a unique solution a_{22} entry must be non-zero at this step, thus the condition is

$ad-bc \neq 0$, or $ad \neq bc$. //