1. Consider the base case $T_1(\vec{v}) = A_1\vec{v}$. The composition $T_2(T_1(\vec{v})) = A_2(A_1\vec{v})$ $= A_2A_1\vec{v}$.

Suppose the composition formula applies to the first n linear transformations i.e. $T(\vec{v}) = T_n(T_{n-1}(T_{n-2}(\cdots(T_2(T_1(\vec{v})))))) = A_nA_{n-1}\cdots A_2A_1$. We wish to show it it true for the next composition $T_{n+1}(T(\vec{v})) = T_{n+1}(T_n(\cdots T_2(T_1(\vec{v}))))) = A_{n+1}(A_n(A_{n-1}\cdots A_2A_1))$.

Thus it has been shown. //

2. Let $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$

a. $\vec{u}\cdot\vec{v} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}^T \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = [u_1\ u_2\ u_3]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = u_1v_1 + u_2v_2 + u_3v_3$ by definition of the dot product. Using the property of commutativity of multiplication of real numbers we can rearrange to get $v_1u_1 + v_2u_2 + v_3u_3 = [v_1\ v_2\ v_3]\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}^T \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \vec{v}\cdot\vec{u}$. //

b. $(\vec{u}+\vec{v})\cdot\vec{w} = \left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right)\cdot\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} u_1+v_1 \\ u_2+v_2 \\ u_3+v_3 \end{bmatrix}^T \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = [u_1+v_1\ \ u_2+v_2\ \ u_3v_3]\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$

$= (u_1+v_1)w_1 + (u_2+v_2)w_2 + (u_3+v_3)w_3 = u_1w_1 + v_1w_1 + u_2w_2 + v_2w_2 + u_3w_3 + v_3w_3$

by definition of the dot product using distributive property of real numbers

Collecting terms we get $(u_1w_1 + u_2w_2 + u_3w_3) + (v_1w_1 + v_2w_2 + v_3w_3) =$

$[u_1\ u_2\ u_3]\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + [v_1\ v_2\ v_3]\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$ by property of the dot product and this

equals $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}^T \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}^T \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \vec{u}\cdot\vec{w} + \vec{v}+w.$ //

c. for c a real number, $(c\vec{u})\cdot\vec{v} = c\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}^T \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = c[u_1\ u_2\ u_3]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} =$

$[cu_1\ cu_2\ cu_3]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = cu_1v_1 + cu_2v_2 + cu_3v_3$ by property of the dot product