202 Proof Set #4

1. Consider the eigenvalue equation $A\vec{x} = \lambda\vec{x}$. for $A = P$ and $\vec{x} = \vec{q}$. it seems pretty clear that $\lambda = 1$ in this relationship. Then by definition of eigenvalues and eigenvectors, $\lambda = 1$ is the eigenvalue and $\vec{q}$ is an eigenvector for P. //

2. Consider $A\vec{x} = \lambda\vec{x}$. if we take the transpose of both sides we get $(A\vec{x})^T = (\lambda\vec{x})^T \Rightarrow \vec{x}^T A^T = \vec{x}^T \cdot \lambda$. Though the transpose multiplies the vector on the left, λ is still the eigenvalue of A^T and $\vec{x}^T$ is an eigenvector. //

3. Consider $A\vec{x} = \lambda\vec{x}$. if we multiply both sides of the equation by A^{-1} we get $(A^{-1}A)\vec{x} = A^{-1}(\lambda\vec{x})$ or $I\vec{x} = \lambda(A^{-1}\vec{x})$. Since we can divide by λ if $\lambda \neq 0$ (which is required if A is invertible) we get $A^{-1}\vec{x} = \frac{1}{\lambda}\vec{x}$. This $1/\lambda$ is an eigenvalue of A^{-1}. //

4. If A and B are similar, then if $A\vec{x} = \lambda x$ and $A = PBP^{-1}$, we have $(PBP^{-1})\vec{x} = \lambda\vec{x}$. Multiplying both sides by P^{-1} we get $P^{-1}PBP^{-1}\vec{x} = BP^{-1}\vec{x} = P^{-1}(\lambda\vec{x}) = \lambda P^{-1}\vec{x}$. or $B(P^{-1}\vec{x}) = \lambda(P^{-1}\vec{x})$ thus $P^{-1}\vec{x}$ is the eigenvector of **B**, but λ is the corresponding eigenvalue of B, which is the same eigenvalue as A. //

5. To show that $A = \begin{bmatrix} 1 & -6 \\ 2 & -6 \end{bmatrix}$ is diagonalizable (in real numbers), then A must have distinct eigenvalues, or if the eigenvalue is repeated, it must have a 2-dimensional eigenspace. The characteristic polynomial of A is $(1-\lambda)(-6-\lambda) + 12 = \lambda^2 + 5\lambda - 6 + 12 = \lambda^2 + 5\lambda + 6 = 0$ which implies that $(\lambda+2)(\lambda+3) = 0$ or that $\lambda = -2$, and $\lambda = -3$. Since the eigenvalues are distinct, A is diagonalizable. //