

# 2/2 Homework #10 key

①

1a.  $\langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x) dx$

$\langle \sin(nx), \sin(mx) \rangle = \int_{-\pi}^{\pi} \sin(nx)\sin(mx) dx$

$\sin a \sin b = \frac{1}{2} (\cos(a-b) - \cos(a+b))$

$\frac{1}{2} \int_{-\pi}^{\pi} \cos[(n-m)x] - \cos[(m+n)x] dx$

if  $n \neq m$

$= \frac{1}{2} \left[ \frac{\sin[(n-m)x]}{n-m} - \frac{\sin[(m+n)x]}{m+n} \right]_{-\pi}^{\pi}$

$n, m \in \mathbb{Z}$

$n-m = k \in \mathbb{Z}$

$n+m = p \in \mathbb{Z}$

$\frac{1}{2} \left[ \frac{\sin k\pi}{k} - \frac{\sin p\pi}{p} \right] = 0$  orthogonal

if  $n = m$

$\frac{1}{2} \int_{-\pi}^{\pi} \cos(0) - \cos(2n) dx = \frac{1}{2} \left[ x - \frac{\sin 2n}{2n} \right]_{-\pi}^{\pi} = \pi$  not orthogonal w/ itself

b.  $\cos(nx), \cos(mx)$

$\cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$

$\int_{-\pi}^{\pi} \cos nx \cdot \cos mx dx = \frac{1}{2} \int_{-\pi}^{\pi} \cos(n+m)x + \cos(m-n)x dx$

if  $m \neq n$

$\frac{1}{2} \left[ \frac{\sin(n+m)x}{n+m} + \frac{\sin(m-n)x}{n-m} \right]_{-\pi}^{\pi}$

$n \in \mathbb{Z}, m \in \mathbb{Z}$

$n \pm m \in \mathbb{Z}, n+m = k \in \mathbb{Z}$   
 $n-m = p \in \mathbb{Z}$

$\frac{1}{2} \left[ \frac{\sin k\pi}{k} + \frac{\sin p\pi}{p} \dots \right] = 0$  orthogonal if  $m \neq n$

if  $m = n$

$\frac{1}{2} \int_{-\pi}^{\pi} \cos 2n + \cos 0 dx = \frac{1}{2} \left[ \frac{\sin 2nx}{2n} + x \right]_{-\pi}^{\pi} = \pi$  not orthogonal to itself

c.  $\langle \sin nx, \cos mx \rangle$

$\frac{1}{2} (\sin(a+b) + \sin(a-b))$

$\int_{-\pi}^{\pi} \sin(nx) \cos(mx) dx = \frac{1}{2} \int_{-\pi}^{\pi} \sin(n+m)x + \sin(n-m)x dx = n \neq m$

$\frac{1}{2} \left[ \frac{-\cos(n+m)x}{n+m} - \frac{\cos(n-m)x}{n-m} \right]_{-\pi}^{\pi} = -\frac{1}{2} \left[ \frac{\cos k\pi}{k} + \frac{\cos p\pi}{p} - \frac{\cos k\pi}{k} - \frac{\cos p\pi}{p} \right] = 0$   
 $n, m \in \mathbb{Z} \Rightarrow n+m, n-m \in \mathbb{Z}$   
 $2p$