

2/2 Homework #11 key

①

$$1. f(t) = \begin{cases} 0 & t < 1 \\ t^2 - 2t + 2 & t \geq 1 \end{cases} \quad \begin{matrix} t^2 - 2t + 2 = \\ (t-1)^2 + 1 \end{matrix}$$

$$[(t-1)^2 + 1]u_1(t) \text{ or } [(t-1)^2 + 1]u(t-1)$$

$$\Rightarrow \mathcal{L}\{f(t)\} = e^{-s} \left(\frac{2}{s^3} + \frac{1}{s} \right)$$

$$2a. \frac{3!}{(s-2)^4} = t^3 e^{2t}$$

$$b. \frac{(s-2)e^{-s}}{s^2-4s+3} = e^{-s} \left[\frac{s-2}{(s-3)(s-1)} \right] = e^{-s} \left[\frac{A}{s-3} + \frac{B}{s-1} \right]$$

$$As - A + Bs - 3B$$

$$\begin{matrix} A+B=1 \\ -A-3B=-2 \end{matrix} \quad \left[\begin{array}{cc|c} 1 & 1 & 1 \\ -1 & -3 & -2 \end{array} \right] \quad \begin{matrix} A=1/2 \\ B=1/2 \end{matrix}$$

$$\frac{1}{2} e^{-s} \left[\frac{1}{s-3} + \frac{1}{s-1} \right] = \frac{1}{2} u_1(t) [e^{3(t-1)} + e^{t-1}]$$

$$c. \frac{e^{-2s}}{s^2-4} = \frac{e^{-2s}}{2} \left[\frac{2}{s^2-4} \right] = \frac{1}{2} u_2(t) \sinh(2(t-2))$$

$$= \frac{1}{2} u_2(t) \sinh(2t-4)$$

$$3. \mathcal{L}\{f(t)\} \text{ for } f(t) = \int_0^t (t-\tau)^2 \cos 2\tau d\tau$$

$$\mathcal{L}\{f(t)\} = \frac{2}{s^3} \cdot \frac{s}{s^2+4} = \frac{2}{s^2(s^2+4)} \quad \begin{matrix} g=t^2 & h=\cos 2t \end{matrix}$$

$$b. \mathcal{L}^{-1}\{F(s)\} \quad F(s) = \frac{s}{(s+1)(s^2+4)} = \frac{1}{s+1} \cdot \frac{s}{s^2+4}$$

$$f(t) = \int_0^t e^{-(t-\tau)} \sin(2\tau) d\tau$$

$$4.a. y'' + 2y' + 2y = \sin \alpha t \quad y(0) = 0, y'(0) = 0$$

$$s^2 Y(s) + 2s Y(s) + 2Y(s) = \frac{\alpha}{s^2 + \alpha^2}$$