

# 2/2 Homework #12 key

①

1a.  $u'' + \frac{1}{2}u' + 2u = 0$  let  $u = x_1$   $u' = x_1' = x_2$   $x_2' = u''$

$$x_2' + \frac{1}{2}x_2 + 2x_1 = 0 \Rightarrow \begin{cases} x_1' = x_2 \\ x_2' = -2x_1 - \frac{1}{2}x_2 \end{cases}$$

$$\vec{X}' = \begin{bmatrix} 0 & 1 \\ -2 & -\frac{1}{2} \end{bmatrix} \vec{X}$$

b.  $u^{IV} - u = 0$   $u = x_1$   $u' = x_1' = x_2$   $u'' = x_2' = x_3$   $u''' = x_3' = x_4$   $u^{IV} = x_4'$

$$x_4' - x_1 = 0 \quad \begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = x_4 \\ x_4' = x_1 \end{cases} \Rightarrow \vec{X}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \vec{X}$$

c.  $t^2 u'' + t u' + (t^2 - \frac{1}{4})u = 0$   $u = x_1$   $u' = x_1' = x_2$   $x_2' = u''$

$$\begin{cases} t^2 x_2' + t x_2 + (t^2 - \frac{1}{4})x_1 = 0 \\ x_2' = -\frac{1}{t}x_2 + (1 - \frac{1}{4t^2})x_1 \end{cases} \quad \begin{cases} x_1' = x_2 \\ x_2' = (\frac{1}{4t^2} - 1)x_1 - \frac{1}{t}x_2 \end{cases}$$

$$\vec{X}' = \begin{bmatrix} 0 & 1 \\ \frac{1}{4t^2} - 1 & -\frac{1}{t} \end{bmatrix} \vec{X}$$

d.  $u'' + \frac{1}{4}u' + 4u = 2 \cos 3t$   $u(0) = 1$ ,  $u'(0) = -2$

$$\begin{cases} x_1' = x_2 \\ x_2' = -\frac{1}{4}x_2 - 4x_1 + 2 \cos 3t \end{cases} \quad \begin{cases} u = x_1, x_1' = x_2 = u' \\ x_2' = u'' \end{cases}$$

$$\vec{X}' = \begin{bmatrix} 0 & 1 \\ -4 & -\frac{1}{4} \end{bmatrix} \vec{X} + \begin{bmatrix} 0 \\ 2 \cos 3t \end{bmatrix} \quad \vec{X}(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

2a.  $x_1' = 3x_1 - 2x_2$   $x_2' = 2x_1 - 2x_2$

$$\frac{x_1' - 3x_1}{-2} = x_2 = -\frac{1}{2}x_1' + \frac{3}{2}x_1$$

$$x_2' = -\frac{1}{2}x_1'' + \frac{3}{2}x_1'$$

$$\begin{cases} r^2 - r - 2 = 0 \\ (r-2)(r+1) = 0 \\ r = 2, r = -1 \end{cases}$$

$$-\frac{1}{2}x_1'' + \frac{3}{2}x_1' = 2x_1 - 2(\frac{1}{2}x_1' + \frac{3}{2}x_1)$$

$$-2(-\frac{1}{2}x_1'' + \frac{3}{2}x_1' = 2x_1 + x_1' - 3x_1)$$

$$x_1'' - 3x_1' = -4x_1 + 2x_1' + 6x_1$$

$$x_1'' - x_1' - 2x_1 = 0 \Rightarrow u'' - u' - 2u = 0$$

$$u(0) = 3$$

$$u'(0) = 3(3) - 2(1) = 8$$

$$u(t) = c_1 e^{2t} + c_2 e^{-t}$$



# 2/2 Homework #12 key cont'd

(2)

2a cont'd

$$u'(t) = 2c_1 e^{2t} + (-1)c_2 e^{-t}$$

$$c_1 + c_2 = 3$$

$$2c_1 - c_2 = 8$$

$$3c_1 = 11$$

$$c_1 = \frac{11}{3}$$

$$c_2 = 3 - \frac{11}{3} = -\frac{2}{3}$$

$$u(t) = \frac{11}{3} e^{2t} - \frac{2}{3} e^{-t}$$



$$2b. x_1' = 2x_2 \Rightarrow \frac{1}{2}x_1' = x_2 \Rightarrow \frac{1}{2}x_1'' = x_2'$$

$$x_2' = -2x_1$$

$$2\left(\frac{1}{2}x_1'' = -2x_1\right) \Rightarrow x_1'' = -4x_1 \Rightarrow x_1'' + 4x_1 = 0$$

$$r^2 + 4 = 0$$

$$r = \pm 2i$$

$$u(t) = c_1 \cos 2t + c_2 \sin 2t \rightarrow c_1(1) + c_2(0) = 3$$

$$c_1 = 3$$

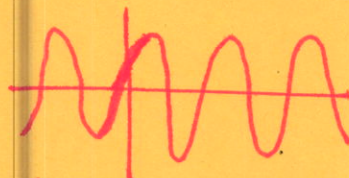
$$u'(t) = -2c_1 \sin 2t + 2c_2 \cos 2t \Rightarrow -2c_1(0) + 2c_2(1) = 8$$

$$c_2 = 4$$

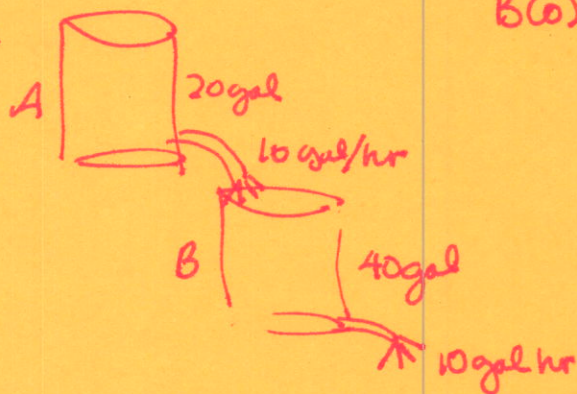
$$u(t) = 3 \cos 2t + 4 \sin 2t$$

$$A(0) = 500$$

$$B(0) = 500$$



3.



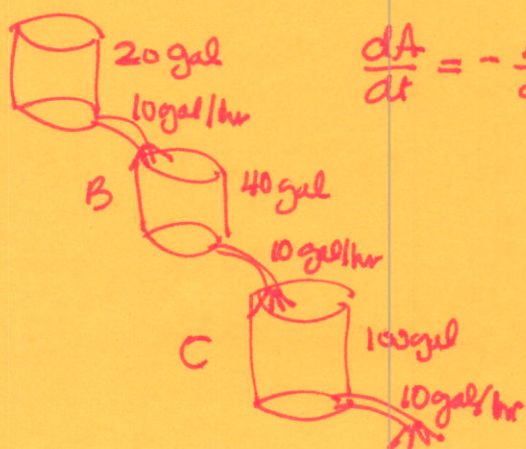
$$\frac{dA}{dt} = \text{rate in} - \text{rate out}$$

$$\frac{dA}{dt} = 0 - \frac{A}{20} \cdot \frac{10}{\text{hr}} = -\frac{A}{2}$$

$$\frac{dB}{dt} = \frac{A}{2} - \frac{B}{40} \cdot \frac{10}{\text{hr}} = \frac{A}{2} - \frac{B}{4}$$

$$\begin{bmatrix} A \\ B \end{bmatrix}' = \begin{bmatrix} -1/2 & 0 \\ 1/2 & 1/4 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

4.



$$\frac{dA}{dt} = -\frac{A}{2}$$

$$\frac{dB}{dt} = \frac{A}{2} - \frac{B}{4}$$

$$\frac{dC}{dt} = \frac{B}{4} - \frac{C}{100} \cdot \frac{10}{\text{hr}} = \frac{B}{4} - \frac{C}{10}$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix}' = \begin{bmatrix} -1/2 & 0 & 0 \\ 1/2 & -1/4 & 0 \\ 0 & 1/4 & -1/10 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$