

212 Homework #1 Key

①

a. $y = te^t; y' = e^t + te^t$ (product rule)

b. $y = e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2}; y' = 2te^{t^2} \int_0^t e^{-s^2} ds + e^{t^2} \cdot e^{-t^2} + 2te^{t^2}$
 $= 2te^{t^2} \int_0^t e^{-s^2} ds + 1 + 2te^{t^2}$ (product rule + chain rule + 2nd fundamental theorem of calculus)

c. $y = e^t \cos 2t; y' = e^t \cos 2t - 2e^t \sin 2t$ (product rule + chain rule)

d. $y = (\cos t) \ln(\cos t) + t \sin t; y' = -(\sin t) \ln(\cos t) + (\cos t) \frac{-\sin t}{\cos t}$
 $+ \sin t + t \cos t = -(\sin t) \ln(\cos t) - \sin t + \sin t + t \cos t$
 $= -(\sin t) \ln(\cos t) + t \cos t$ (product rule + chain rule)

2.a. $y_2(t) = \cosh t$
 $y_2' = \sinh t$
 $y_2'' = \cosh t$

$y_1(t) = e^t$
 $y_1' = e^t$
 $y_1'' = e^t$

$y'' - y = 0$
 $\cosh t - \cosh t = 0 \checkmark$
 $e^t - e^t = 0 \checkmark$

b. $y_1(t) = t/3$
 $y_1' = 1/3$
 $y_1'' = 0$
 $y_1''' = 0$
 $y_1^{(4)} = 0$

$y_2(t) = \frac{t}{3} e^{-t}$
 $y_2' = \frac{1}{3} e^{-t}$
 $y_2'' = -e^{-t}$
 $y_2''' = -e^{-t}$
 $y_2^{(4)} = e^{-t}$

$y^{(4)} + 4y''' + 3y = t$
 $0 + 0(4) + 3(t/3) = t \checkmark$
 $e^{-t} + 4(-e^{-t}) + 3(\frac{t}{3} + e^{-t}) =$
 $e^{-t} - 4e^{-t} + 3e^{-t} + t = t \checkmark$

c. $y_1(t) = 3t + t^2$
 $y_1' = 3 + 2t^2$

$ty' - y = t^2$
 $t(3 + 2t^2) - (3t + t^2) =$
 $3t + 2t^2 - 3t - t^2 = t^2 \checkmark$