

212 Homework #2 Key

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$$1a. A\vec{u} = \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \cdot 5 + (-1)(-1) \\ 4 \cdot 5 + (-1)(-1) \end{bmatrix} = \begin{bmatrix} 10+1 \\ 20-1 \end{bmatrix} = \begin{bmatrix} 11 \\ 19 \end{bmatrix}$$

$$b. B\vec{v} = \begin{bmatrix} i & 1+i \\ 2-i & -3 \end{bmatrix} \begin{bmatrix} 3i \\ 1-4i \end{bmatrix} = \begin{bmatrix} i(3i) + (1+i)(1-4i) \\ (2-i)(3i) + (-3)(1-4i) \end{bmatrix} = \begin{bmatrix} 3i^2 + 1 - 4i + i - 4i^2 \\ 6i - 3i^2 - 3 + 12i \end{bmatrix} = \begin{bmatrix} -3 + 1 - 3i + 4 \\ 6i + 3 - 3 + 12i \end{bmatrix} = \begin{bmatrix} 2 - 3i \\ 18i \end{bmatrix}$$

$$c. C\vec{x} = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & 4 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 - 1 - 6 \\ 2 - 1 + 12 \\ -1 + 0 + 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 13 \\ 2 \end{bmatrix}$$

$$d. AB = \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} i & 1+i \\ 2-i & -3 \end{bmatrix} = \begin{bmatrix} 2i - 2 + 2i + 3 & 2 + 2i - 3 \\ 4i + 2 - i & 4 + 4i - 3 \end{bmatrix} = \begin{bmatrix} -2 + 3i & 5 + 2i \\ 2 + 3i & 1 + 4i \end{bmatrix}$$

$$e. BA = \begin{bmatrix} i & 1+i \\ 2-i & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2i + 4 + 4i & -i + 1+i \\ 4 - 2i - 12 & -2i - 3 \end{bmatrix} = \begin{bmatrix} 4 + 6i & 1 \\ -8 - 2i & -5 + i \end{bmatrix}$$

$$f. B^* = \overline{B}^T = \overline{\begin{bmatrix} i & 1+i \\ 2-i & -3 \end{bmatrix}}^T = \begin{bmatrix} -i & 1-i \\ 2+i & -3 \end{bmatrix}^T = \begin{bmatrix} -i & 2+i \\ 1-i & -3 \end{bmatrix}$$

$$g. C^T = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & 4 \\ -1 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 1 & 0 \\ -2 & 4 & 1 \end{bmatrix}$$

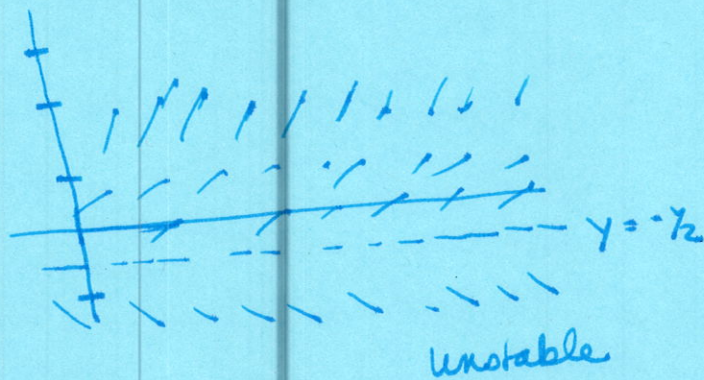
$$2a. y' = 1 + 2y = 2(\frac{1}{2} + y)$$

$$0 = \frac{1}{2} + y$$

$$y = -\frac{1}{2}$$

$$y > -\frac{1}{2} \quad +$$

$$y < -\frac{1}{2} \quad -$$



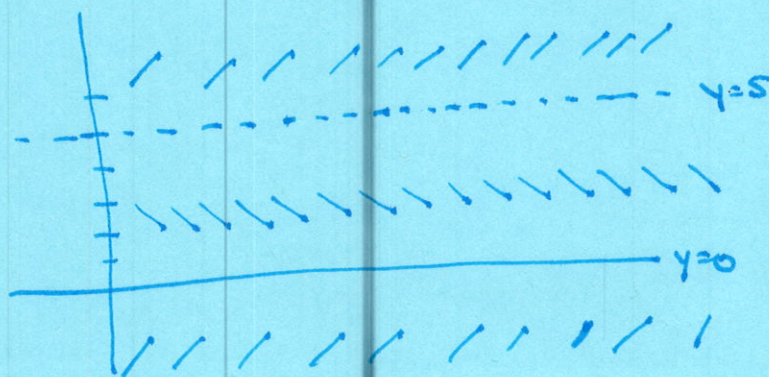
$$b. y' = -y(5-y) = y(y-5)$$

$$0 = -y(5-y)$$

$$y = 0, y = 5$$

$y = 0$ stable

$y = 5$ unstable



H2 Homework #2 Key Cont'd

(2)

2c. $y' = y(y-2)^2$ $y=0, y=2$

$y=0$ unstable

$y=2$ semistable



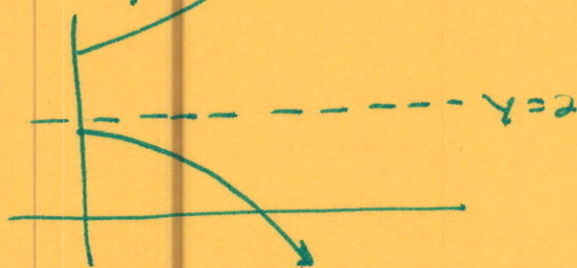
3. See attached

4. a. equilibrium at $y=2$

$y' = y-2$ or $y' = 2-y$
 $y=0$ - +

$y' = y-2$ matches signs

Signs $y=0$



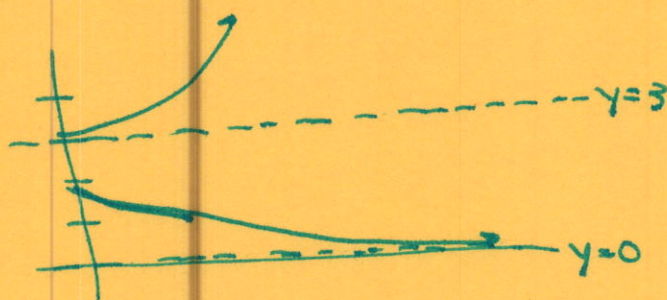
b. equilibrium at $y=0, y=3$

$y' = y(y-3)$ or $y' = -y(y-3)$

$y=1$ - +
 $y=4$ + -

$y' = y(y-3)$ matches signs

Signs



5 linear

$\frac{d^2y}{dt^2}$ is y''' , $\frac{dy}{dt} = y'$, 3rd y term is just y

all nonlinear components are in t which doesn't count for linearity w/ ODEs

6a. $y' = \frac{x^2}{y(1+x^3)^4} \Rightarrow y dy = \frac{x^2}{(1+x^3)^4} dx$ $u = 1+x^3$
 $du = 3x^2$
 $\frac{1}{3} du = x^2$

$\int y dy = \int \frac{1}{3} u^{-4} du \Rightarrow \frac{1}{2} y^2 = -\frac{1}{9} u^{-3} + C$

$y^2 = -\frac{2}{9} \left(\frac{1}{(1+x^3)^3} \right) = \frac{-2}{9(1+x^3)^3} + C$