212 Homework #3 Key

1a. $y' - 2y = 3e^t$ $\quad \mu = e^{\int -2\,dt} = e^{-2t}$

$e^{-2t}y' - 2e^{-2t}y = 3e^{-t} \Rightarrow \int (e^{-2t}y)' = \int 3e^{-t}$

$e^{2t} \cdot e^{-2t}y = (-3e^{-t} + C) \cdot e^{2t}$

$y = -3e^t + Ce^{2t}$

b. $\dfrac{ty' - y = -t^2e^{-t}}{t}$ $\quad t>0$

$y' - \frac{1}{t}y = te^{-t}$ $\quad \mu = e^{\int -\frac{1}{t}dt} = e^{-\ln t} = e^{\ln(1/t)} = 1/t$

$\frac{1}{t}y' - \frac{1}{t^2}y = e^{-t}$

$\int (\frac{1}{t}y)' = \int e^{-t} \Rightarrow \left(\frac{1}{t}y = -e^{-t} + C\right)t$

$y = -te^{-t} + Ct$

c. $\dfrac{t^3y' + 4t^2y = e^{-t}}{t^3}$; $y(-1) = 0$ $\quad t<0$

$y' + \frac{4}{t}y = t^{-3}e^{-t}$ $\quad \mu = e^{\int \frac{4}{t}dt} = e^{4\ln t} = e^{\ln t^4} = t^4$

$t^4y' + 4t^3y = te^{-t} \Rightarrow \int (t^4y)' = \int te^{-t}\,dt$ $\quad u = t,\ du = dt,\ dv = e^{-t},\ v = -e^{-t}$

$+ \int e^{-t}\,dt$

$t^4y = -te^{-t} - e^{-t} + C$

$y = \frac{-1}{t^3e^t} - \frac{1}{t^4e^t} + \frac{C}{t^4}$ $\quad 0 = \frac{-1}{(-1)^3e^{-1}} - \frac{1}{(-1)^4e^{-1}} + \frac{C}{(-1)^4}$

$0 = \frac{1}{e^{-1}} - \frac{1}{e^{-1}} + \frac{C}{1} \Rightarrow C = 0$

$y = $ [illegible] $- \frac{1}{t^3e^t} - \frac{1}{t^4e^t}$

d. $\dfrac{ty' + 2y = \sin t}{t}$ $\quad t>0$ $\quad \mu = e^{\int \frac{2}{t}dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

$y' + \frac{2}{t}y = \frac{\sin t}{t}$

Homework #3 Key cont'd

1d. cont'd

$$t^2y' + 2ty = t\sin t \Rightarrow \int (t^2y)' = \int t\sin t$$

$$t^2y = \int t\sin t\,dt \qquad \begin{matrix} u = t & dv = \sin t\,dt \\ du = dt & v = -\cos t \end{matrix}$$

$$= -t\cos t + \int \cos t\,dt$$

$$t^2y = -t\cos t + \sin t + C$$

$$y = -\frac{\cos t}{t} + \frac{\sin t}{t^2} + \frac{C}{t}$$

e. $y' - 2y = e^{2t}$, $y(0) = 2$ $\qquad \mu = e^{\int -2\,dt} = e^{-2t}$

$$e^{-2t}y' - 2e^{-2t}y = 1 \Rightarrow \left(e^{-2t}y\right)' = \int 1\,dt$$

$$e^{-2t}y = t + C$$

$$y = te^{2t} + Ce^{2t} \qquad 2 = 0e^0 + Ce^0 \Rightarrow$$

$$2 = C$$

$$y = te^{2t} + 2e^{2t}$$

2. You should obtain the same solutions as in #1.

3. $S' = rS + k$ $\qquad S(0) = 8000$ $\qquad r = .1$

$S(3) = 0$

$S' = .1S + k$

$S' - \frac{1}{10}S = k \qquad \mu = e^{\int -\frac{1}{10}dt} = e^{-\frac{t}{10}}$

$$e^{-\frac{t}{10}}S' - \frac{1}{10}e^{-\frac{t}{10}}S = ke^{-\frac{t}{10}}$$

$$\int \left(e^{-\frac{t}{10}}S\right)' = \int ke^{-\frac{t}{10}}\,dt \Rightarrow Se^{-\frac{t}{10}} = -10ke^{-\frac{t}{10}} + C \qquad)*e^{\frac{t}{10}}$$

$$S = -10k + Ce^{\frac{t}{10}}$$

$$8000 = -10k + C$$
$$0 = -10k + Ce^{3/10}$$

$$\begin{matrix} 8000 = -10k + C \\ 0 = 10k - Ce^{3/10} \end{matrix}$$

$$8000 = C(1 - e^{3/10}) \Rightarrow C = \frac{8000}{1 - e^{3/10}} \approx -22{,}866.37$$

$$\frac{8000 + 22{,}866.37}{-10} \approx -3086.64 = k$$