

212 Homework #3 Key

a.  $y' - 2y = 3e^t$

$\mu = e^{\int -2 dt} = e^{-2t}$

$e^{-2t} y' - 2e^{-2t} y = 3e^{-t} \Rightarrow \int (e^{-2t} y)' = \int 3e^{-t}$

$e^{2t} \cdot e^{-2t} y = (-3e^{-t} + C) \cdot e^{2t}$

$y = -3e^t + Ce^{2t}$

b.  $\frac{ty' - y}{t} = -te^{-t} \quad t > 0$

$y' - \frac{1}{t}y = -te^{-t}$

$\mu = e^{\int -\frac{1}{t} dt} = e^{-\ln t} = e^{\ln(1/t)} = 1/t$

$\frac{1}{t}y' - \frac{1}{t^2}y = e^{-t}$

$\int (\frac{1}{t}y)' = \int e^{-t} \Rightarrow (\frac{1}{t}y = -e^{-t} + C) t$

$y = -te^{-t} + Ct$

c.  $\frac{t^3 y' + 4t^2 y}{t^3} = e^{-t}; \quad y(-1) = 0 \quad t < 0$

$y' + \frac{4}{t}y = t^{-3}e^{-t}$

$\mu = e^{\int \frac{4}{t} dt} = e^{4 \ln t} = e^{\ln t^4} = t^4$

$t^4 y' + 4t^3 y = te^{-t} \Rightarrow \int (t^4 y)' = \int te^{-t} dt$

$u = t \quad dv = e^{-t}$   
 $du = dt \quad v = -e^{-t}$   
 $+ \int e^{-t} dt$

$t^4 y = -te^{-t} - e^{-t} + C$

$y = \frac{-1}{t^3 e^t} - \frac{1}{t^4 e^t} + \frac{C}{t^4}$

$0 = \frac{-1}{(-1)^3 e^{-1}} - \frac{1}{(-1)^4 e^{-1}} + \frac{C}{(-1)^4}$

$0 = \frac{1}{e} - \frac{1}{e^{-1}} + \frac{C}{1} \Rightarrow C = 0$

$y = \frac{-1}{t^3 e^t} - \frac{1}{t^4 e^t}$

d.  $\frac{ty' + 2y}{t} = \frac{\sin t}{t} \quad t > 0$

$y' + \frac{2}{t}y = \frac{\sin t}{t}$

$\mu = e^{\int \frac{2}{t} dt} = e^{2 \ln t} = e^{\ln t^2} = t^2$



1d. cont'd

$$t^2 y' + 2ty = t \sin t \Rightarrow \int (t^2 y)' = \int t \sin t$$

$$t^2 y = \int t \sin t dt \quad \begin{array}{l} u=t \quad dv=\sin t \\ du=dt \quad v=-\cos t \end{array}$$

$$= -t \cos t + \int \cos t dt$$

$$t^2 y = -t \cos t + \sin t + C$$

$$y = -\frac{\cos t}{t} + \frac{\sin t}{t^2} + \frac{C}{t}$$

e.  $y' - 2y = e^{2t}, y(0) = 2 \quad \mu = e^{\int -2dt} = e^{-2t}$

$$e^{-2t} y' - 2e^{-2t} y = 1 \Rightarrow (e^{-2t} y)' = \int 1 dt$$

$$e^{-2t} y = t + C$$

$$y = te^{2t} + Ce^{2t}$$

$$2 = 0e^0 + Ce^0 \Rightarrow 2 = C$$

$$y = te^{2t} + 2e^{2t}$$

2. you should obtain the same solutions as in #1.

3.  $S' = rS + k$   
 $S' = .1S + k$   
 $S' - \frac{1}{10}S = k$

$S(0) = 8000 \quad r = .1$   
 $S(3) = 0$   
 $\mu = e^{\int -\frac{1}{10} dt} = e^{-\frac{t}{10}}$

$$e^{-\frac{t}{10}} S' - \frac{1}{10} e^{-\frac{t}{10}} S = k e^{-\frac{t}{10}}$$

$$\int (e^{-\frac{t}{10}} S)' = \int k e^{-\frac{t}{10}} dt \Rightarrow S e^{-\frac{t}{10}} = -10k e^{-\frac{t}{10}} + C \quad ) \cdot e^{\frac{t}{10}}$$

$$S = -10k + C e^{\frac{t}{10}}$$

$$\begin{array}{l} 8000 = -10k + C \\ 0 = -10k + C e^{3/10} \end{array}$$

$$\begin{array}{l} 8000 = -10k + C \\ 0 = 10k - C e^{3/10} \end{array}$$

$$8000 = C(1 - e^{3/10}) \Rightarrow C = \frac{8000}{1 - e^{3/10}} \approx -22,866.37$$

$$\frac{8000 + 22,866.37}{-10} = -3086.64 = k$$