

212 Homework #4 key

- a. See attached
- b. See attached
- c. See attached
- d. See attached

2a. $y' = 2y - 1 \Rightarrow y' = 2(y - 1/2) \Rightarrow \int \frac{dy}{y - 1/2} = \int 2 dt \Rightarrow \ln(y - 1/2) = 2t + C$
 $\Rightarrow y - 1/2 = Ae^{2t}$
 $y(t) = Ae^{2t} + 1/2$
 $1 = Ae^0 + 1/2 \Rightarrow 1/2 = A$
 $y(t) = 1/2 e^{2t} + 1/2$
 $y(1/2) = 1/2 e^{(1)} + 1/2 \approx 1.8591$

$$\begin{aligned} e^{2t+C} &= e^C \cdot e^{2t} \\ A &= e^C \\ y(0) &= 1 \end{aligned}$$

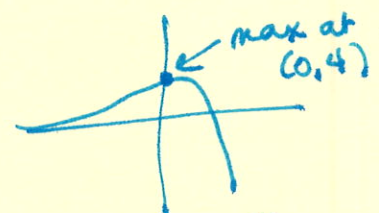
The solutions are more accurate the more steps are used
 pretty good estimate

b. $y' = y(3 - ty) \quad y(0) = 2$
 $y' = 3y - ty^2 \Rightarrow y' - 3y = -ty^2 \quad (1-u)y' = (1-2)y^2 = -1y^2$
 $-y^{-2}y' + 3y^{-1} = t \Rightarrow z = y^{-1} \quad z' = -y^{-2}y'$
 $z' + 3z = t \Rightarrow \mu = e^{\int 3 dt} = e^{3t} \Rightarrow e^{3t}z' + 3e^{3t}z = e^{3t}t \Rightarrow$
 $\int (e^{3t}z)' = \int e^{3t}t$
 $u=t \quad dv=e^{3t} \quad \frac{1}{3}te^{3t} - \frac{1}{3}\int e^{3t} dt$
 $du=dt \quad v=1/3e^{3t}$
 $e^{3t}z = \frac{1}{3}te^{3t} - \frac{1}{9}e^{3t} + C \Rightarrow z = \frac{1}{3}t - \frac{1}{9} + Ce^{-3t}$
 $\Rightarrow \frac{1}{y} = \frac{1}{3}t - \frac{1}{9} + Ce^{-3t} \Rightarrow \frac{1}{2} = \frac{1}{3}(0) - \frac{1}{9} + Ce^0$
 $1/2 + 1/9 = C \Rightarrow C = 7/18$

$\frac{1}{y} = \frac{1}{3}t - \frac{1}{9} + \frac{7}{18}e^{-3t} \quad t=1/2 \quad y(t) = \frac{1}{\frac{1}{3}(1/2) - \frac{1}{9} + \frac{7}{18}e^{-3/2}} \approx 7.026$

nonlinear and kinda off here.

3.a. $6y'' - 5y' + y = 0 \quad y(0) = 4, y'(0) = 0$
 $6r^2 - 5r + 1 = 0 \quad (3r-1)(2r-1) = 0 \quad r = 1/3, r = 1/2$
 $y(t) = c_1 e^{1/3 t} + c_2 e^{1/2 t} \quad y' = 1/3 c_1 e^{1/3 t} + 1/2 c_2 e^{1/2 t}$
 $4 = c_1 + c_2$
 $0 = 1/3 c_1 + 1/2 c_2$
 $y(t) = 12e^{1/3 t} - 8e^{1/2 t}$



$c_1 = 12, c_2 = -8$

goes to $-\infty$ as $t \rightarrow \infty$

b. $2y'' - 3y' + y = 0 \quad y(0) = 2, y'(0) = 1/2$
 $2r^2 - 3r + 1 = 0 \quad (2r-1)(r-1) = 0 \quad r = 1/2, r = 1$
 $y(t) = c_1 e^{1/2 t} + c_2 e^t$

Homework #4 Key cont'd

3b (cont'd) $y'(t) = \frac{1}{2}c_1 e^{\frac{1}{2}t} + c_2 e^t$

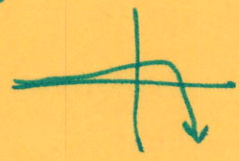
$c_1 + c_2 = 2$
 $\frac{1}{2}c_1 + c_2 = \frac{1}{2}$

$\Rightarrow \left[\begin{array}{c|c} 1 & 1 \\ \hline \frac{1}{2} & 1 \end{array} \middle| \begin{array}{c} 2 \\ \frac{1}{2} \end{array} \right] \Rightarrow \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array} \middle| \begin{array}{c} 3 \\ -1 \end{array} \right]$

$c_1 = 3, c_2 = -1$

$y(t) = 3e^{\frac{1}{2}t} - e^t$

max at $\approx (1.81, 2.25)$



graph $\rightarrow 0$ as $t \rightarrow \infty$

3c. $y'' + 4y' + 5y = 0$

$y(0) = 1, y'(0) = 0$

$r^2 + 4r + 5 = 0$

does not factor

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$r = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$

$e^{(-2 \pm i)t} = e^{-2t} \cdot e^{\pm it} = e^{-2t} (\cos t \pm i \sin t) \Rightarrow$

$y(t) = c_1 e^{-2t} \cos t + c_2 e^{-2t} \sin t$

$y' = -2c_1 e^{-2t} \cos t - c_1 e^{-2t} \sin t - 2c_2 e^{-2t} \sin t + c_2 e^{-2t} \cos t$

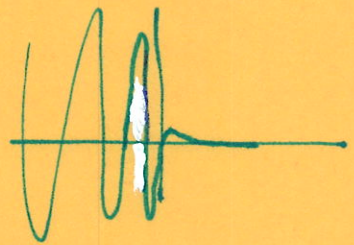
$c_1(0) + c_2(0) = 1 \Rightarrow c_1 = 1$

$-2(1)(1)(1) - (1)(1)(0) - 2c_2(1)(0) + c_2(1)(1) = 0$

$-2 + c_2 = 0 \Rightarrow c_2 = 2$

$y(t) = e^{-2t} \cos t + 2e^{-2t} \sin t$

infinite # of critical points
 $\rightarrow 0$ as $t \rightarrow \infty$



3d. $y'' + 4y' + 4y = 0$ $y(-1) = 2$ $y'(-1) = 1$

$r^2 + 4r + 4 = 0$ $(r+2) = 0$ $r = -2$

$y(t) = c_1 e^{-2t} + c_2 t e^{-2t}$

$y'(t) = -2c_1 e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t}$

$c_1 e^2 + c_2(-1)e^2 = 2 \Rightarrow c_1 - c_2 = \frac{2}{e^2}$

$\Rightarrow 2c_1 - 2c_2 = \frac{4}{e^2}$

$-2c_1 e^2 + c_2 e^2 - 2c_2(-1)e^2 = 1 \Rightarrow -2c_1 + 3c_2 = \frac{1}{e^2}$

$-2c_1 + 3c_2 = \frac{1}{e^2}$

$c_1 = \frac{2}{e^2} - \frac{5}{e^2} = -\frac{3}{e^2}$

$c_2 = \frac{5}{e^2}$

$y(t) = \frac{3}{e^2} e^{-2t} + \frac{5}{e^2} t e^{-2t} = 3e^{-2t-2} + 5te^{-2t-2}$

$= 3e^{-2(t+1)} + 5te^{-2(t+1)}$

as $t \rightarrow \infty, y \rightarrow 0$

max at $\approx (-0.1, .4132)$

