

# 2/2 Homework #6 Key

①

1.a.  $\begin{cases} 2x + 3y = 12 \\ 4x - y = 10 \end{cases}$

$A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \Rightarrow \det A = -2 - 12 = -14$

$A_1 = \begin{bmatrix} 12 & 3 \\ 10 & -1 \end{bmatrix} \Rightarrow \det A_1 = -12 - 30 = -42$

$A_2 = \begin{bmatrix} 2 & 12 \\ 4 & 10 \end{bmatrix} \Rightarrow \det A_2 = 20 - 48 = -28$

$x_1 = \frac{\det A_1}{\det A} = \frac{-42}{-14} = 3$

$x_2 = \frac{\det A_2}{\det A} = \frac{-28}{-14} = 2$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

b.  $\begin{cases} -x + 5y = 17 \\ 3x - 4y = 12 \end{cases}$

$A = \begin{bmatrix} -1 & 5 \\ 3 & -4 \end{bmatrix} \Rightarrow \det A = 4 - 15 = -11$

$A_1 = \begin{bmatrix} 17 & 5 \\ 12 & -4 \end{bmatrix} \Rightarrow \det A_1 = -68 - 60 = -128$

$A_2 = \begin{bmatrix} -1 & 17 \\ 3 & 12 \end{bmatrix} \Rightarrow \det A_2 = -12 - 51 = -63$

$x_1 = \frac{\det A_1}{\det A} = \frac{-128}{-11} = \frac{128}{11}$

$x_2 = \frac{\det A_2}{\det A} = \frac{-63}{-11} = \frac{63}{11}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 128/11 \\ 63/11 \end{bmatrix}$

c.  $\begin{cases} 5x - y + 2z = 10 \\ 3x + 2y - 4z = 16 \\ -4x - 3y + z = 7 \end{cases}$

$A = \begin{bmatrix} 5 & -1 & 2 \\ 3 & 2 & -4 \\ -4 & -3 & 1 \end{bmatrix} \Rightarrow \det A = -65$

$A_1 = \begin{bmatrix} 10 & -1 & 2 \\ 16 & 2 & -4 \\ 7 & -3 & 1 \end{bmatrix} \Rightarrow \det A_1 = -180$

$A_2 = \begin{bmatrix} 5 & 10 & 2 \\ 3 & 16 & -4 \\ -4 & 7 & 1 \end{bmatrix} \Rightarrow \det A_2 = 520$

$A_3 = \begin{bmatrix} 5 & -1 & 10 \\ 3 & 2 & 16 \\ -4 & -3 & 7 \end{bmatrix} \Rightarrow \det A_3 = 385$

$x_1 = \frac{\det A_1}{\det A} = \frac{-180}{-65} = \frac{36}{13}$

$x_2 = \frac{\det A_2}{\det A} = \frac{520}{-65} = -8$

$x_3 = \frac{\det A_3}{\det A} = \frac{385}{-65} = -\frac{77}{13}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 36/13 \\ -8 \\ -77/13 \end{bmatrix}$

212 Homework #6 cont'd

(2)

1d. 
$$\begin{cases} x+y+z=9 \\ -x+2y-3z=14 \\ 3x-5y-2z=-18 \end{cases} \quad A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & -3 \\ 3 & -5 & -2 \end{bmatrix} \quad \det A = -31$$

$$A_1 = \begin{bmatrix} 9 & 1 & 1 \\ 14 & 2 & -3 \\ -18 & -5 & -2 \end{bmatrix} \quad \det A_1 = -123$$

$$A_2 = \begin{bmatrix} 1 & 9 & 1 \\ -1 & 14 & -3 \\ 3 & -18 & -2 \end{bmatrix} \quad \det A_2 = -205$$

$$A_3 = \begin{bmatrix} 1 & 1 & 9 \\ -1 & 2 & 14 \\ 3 & -5 & -18 \end{bmatrix} \quad \det A_3 = 49$$

$$x_1 = \frac{\det A_1}{\det A} = \frac{-123}{-31} = \frac{123}{31} \quad x_2 = \frac{\det A_2}{\det A} = \frac{-205}{-31} = \frac{205}{31} \quad x_3 = \frac{\det A_3}{\det A} = \frac{49}{-31} = -\frac{49}{31}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 123/31 \\ 205/31 \\ -49/31 \end{bmatrix}$$

2a.  $t^2 y'' + 2ty' - 2y = 0 \quad y_1(t) = t \quad y_2 = vt \quad y_2' = v't + v \quad y_2'' = v'' + 2v'$

$t^2(v'' + 2v') + 2t(v't + v) - 2vt = 0$

$t^2 v'' + 2vt^2 + 2t^2 v' + 2vt - 2vt = 0 \Rightarrow t^2(v'' + 4v') = 0$

$u' = -4u \Rightarrow \int \frac{du}{u} = \int -4 dt \quad u = v' \quad u' = v''$

$\ln u = -4t + C \Rightarrow u = Ae^{-4t} \Rightarrow v' = Ae^{-4t} \Rightarrow v = -\frac{A}{4} e^{-4t} = Be^{-4t}$

$y_2 = e^{-4t} \cdot t$

$$\begin{vmatrix} t & te^{-4t} \\ 1 & e^{-4t} - 4te^{-4t} \end{vmatrix} = te^{-4t} - 4t^2 e^{-4t} - te^{-4t} = -4t^2 e^{-4t}$$
  
 Yes, fundamental set

b.  $(x-1)y'' - xy' + y = 0$

$y_1 = e^x \quad y_2 = ve^x \quad y_2' = v'e^x + ve^x \quad y_2'' = v''e^x + 2v'e^x + ve^x$

$(x-1)(v''e^x + 2v'e^x + ve^x) - x(v'e^x + ve^x) + ve^x = 0 \quad /e^x$

$(x-1)(v'' + 2v' + v) - x(v' + v) + v = 0 \Rightarrow xv'' + 2xv' + vx - v'' - 2v' - v - xv' - vx + v = 0$

$v''(x-1) + v'(x-2) = 0 \quad u = v' \quad u' = v''$

$\frac{du}{dx}(x-1) = -(x-2)u \Rightarrow \frac{du}{u} = \frac{-(x-2)}{x-1} = -1 + \frac{1}{x-1} \quad \begin{matrix} -1 \\ x-1 \end{matrix} \quad \begin{matrix} -x+2 \\ -x+1 \\ 1 \end{matrix}$

$\ln u = -x + \ln(x-1) + C \Rightarrow u = A(x-1)e^{-x} = v' \quad v = \int (x-1)e^{-x} dx$