

# 212 Homework #7 key

(1)

1a.  $1^{1/3}$

$$1 = e^{0\pi i} = e^{2\pi i} = e^{4\pi i}$$

$$1^{1/3} = e^{0\pi/3 i} \rightarrow e^{2\pi/3 i} \rightarrow e^{4\pi/3 i}$$

$$e^{0i} = 1$$

$$e^{2\pi/3 i} = \cos(2\pi/3) + i \sin(2\pi/3) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$e^{4\pi/3 i} = \cos(4\pi/3) + i \sin(4\pi/3) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

b.  $(1-i)^{1/2}$

$$\|1-i\| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$= \sqrt{2} \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) = \sqrt{2} e^{3\pi/4 i} \Rightarrow \sqrt{1-i} = \left( \sqrt{2} e^{3\pi/4 i} \right)^{1/2} =$$

$$\sqrt[4]{2} e^{3\pi/8 i}, \sqrt[4]{2} e^{11\pi/8 i}$$

$$\left( \sqrt{2} e^{11\pi/4 i} \right)^{1/2}$$

$$\sqrt[4]{2} [\cos(3\pi/8) + i \sin(3\pi/8)] = \sqrt[4]{2} \cos(3\pi/8) + i \sqrt[4]{2} \sin(3\pi/8)$$

$$\sqrt[4]{2} [\cos(11\pi/8) + i \sin(11\pi/8)] = \sqrt[4]{2} \cos(11\pi/8) + i \sqrt[4]{2} \sin(11\pi/8)$$

c.  $(-4i)^{1/4} = \left( 4 e^{3\pi/2 i} \right)^{1/4}$

$$(4)^{1/4} = 2^{1/2} = \sqrt{2}$$

$$e^{7\pi/2 i}$$

$$e^{11\pi/2 i}$$

$$e^{15\pi/2 i}$$

$$\sqrt{2} e^{3\pi/8 i} = \sqrt{2} [\cos(3\pi/8) + i \sin(3\pi/8)] = \sqrt{2} \cos(3\pi/8) + i \sqrt{2} \sin(3\pi/8)$$

$$\sqrt{2} e^{7\pi/8 i} = \sqrt{2} [\cos(7\pi/8) + i \sin(7\pi/8)] = \sqrt{2} \cos(7\pi/8) + i \sqrt{2} \sin(7\pi/8)$$

$$\sqrt{2} e^{11\pi/8 i} = \sqrt{2} [\cos(11\pi/8) + i \sin(11\pi/8)] = \sqrt{2} \cos(11\pi/8) + i \sqrt{2} \sin(11\pi/8)$$

$$\sqrt{2} e^{15\pi/8 i} = \sqrt{2} [\cos(15\pi/8) + i \sin(15\pi/8)] = \sqrt{2} \cos(15\pi/8) + i \sqrt{2} \sin(15\pi/8)$$

d.  $(1-\sqrt{3}i)^{1/6}$

$$\|1-\sqrt{3}i\| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\left[ \frac{1}{2} \left( \frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \right]^{1/6} = \left[ \frac{1}{2} e^{11\pi/6 i} \right]^{1/6} \quad \left( \frac{1}{2} \right)^{1/6} = \frac{1}{\sqrt[6]{2}}$$

$$e^{11\pi/6 i} = e^{23\pi/6 i} = e^{35\pi/6 i} = e^{47\pi/6 i} = e^{59\pi/6 i} = e^{71\pi/6 i}$$

d. cont'd

$$\frac{1}{\sqrt{2}} [\cos(\frac{11\pi}{36}) + i \sin(\frac{11\pi}{36})] = \frac{1}{\sqrt{2}} \cos(\frac{11\pi}{36}) + i \frac{1}{\sqrt{2}} \sin(\frac{11\pi}{36})$$

$$\frac{1}{\sqrt{2}} \cos(\frac{23\pi}{36}) + i \frac{1}{\sqrt{2}} \sin(\frac{23\pi}{36}), \frac{1}{\sqrt{2}} \cos(\frac{35\pi}{36}) + i \frac{1}{\sqrt{2}} \sin(\frac{35\pi}{36})$$

$$\frac{1}{\sqrt{2}} \cos(\frac{47\pi}{36}) + i \frac{1}{\sqrt{2}} \sin(\frac{47\pi}{36}), \frac{1}{\sqrt{2}} \cos(\frac{59\pi}{36}) + i \frac{1}{\sqrt{2}} \sin(\frac{59\pi}{36})$$

$$\frac{1}{\sqrt{2}} \cos(\frac{71\pi}{36}) + i \frac{1}{\sqrt{2}} \sin(\frac{71\pi}{36})$$

e.  $(-1)^{\frac{1}{5}} = (e^{i\pi})^{\frac{1}{5}} (e^{3i\pi})^{\frac{1}{5}} (e^{5i\pi})^{\frac{1}{5}} (e^{7i\pi})^{\frac{1}{5}} (e^{9i\pi})^{\frac{1}{5}}$

$$\cos(\frac{\pi}{5}) + i \sin(\frac{\pi}{5}), \cos(\frac{3\pi}{5}) + i \sin(\frac{3\pi}{5}), \cos(\pi) + i \sin(\pi) = -1$$

$$\cos(\frac{7\pi}{5}) + i \sin(\frac{7\pi}{5}), \cos(\frac{9\pi}{5}) + i \sin(\frac{9\pi}{5})$$

2a.  $y^{IV} + 4y''' + 3y = t$   $p(t) = 4$  defined everywhere

b.  $y''' + ty'' + t^2y' + t^3y = \ln t$   $p(t) = t$  defined everywhere  $t > 0$

3a.  $y_1 = 1$   $y_1' = 0$   $y_1'' = 0$   $y_1''' = 0$   $y_1'''' = 0$   $y_1'''' + y_1' = 0$   $0 + 0 = 0 \checkmark$

$y_2 = \cos t$   $y_2' = -\sin t$   $y_2'' = -\cos t$   $y_2''' = \sin t$   $y_2'''' = \cos t$   $y_2'''' + y_2' = 0 \checkmark$

$y_3 = \sin t$   $y_3' = \cos t$   $y_3'' = -\sin t$   $y_3''' = -\cos t$   $y_3'''' = \sin t$   $y_3'''' + y_3' = 0 \checkmark$

$$\begin{vmatrix} 1 & \cos t & \sin t \\ 0 & -\sin t & \cos t \\ 0 & -\cos t & -\sin t \end{vmatrix} = (\sin^2 t + \cos^2 t) = 1 = (1)(1) = 1 \checkmark$$

yes, fundamental set

b.  $y^{IV} + 2y''' + y'' = 0$

$y_1 = 1$   $y_1' = 0$   $y_1'' = 0$   $y_1''' = 0$   $y_1^{IV} = 0$   $0 + 2(0) + 0 = 0 \checkmark$

$y_2 = t$   $y_2' = 1$   $y_2'' = 0$   $y_2''' = 0$   $y_2^{IV} = 0$   $0 + 2(0) + 0 = 0 \checkmark$

$y_3 = e^{-t}$   $y_3' = -e^{-t}$   $y_3'' = e^{-t}$   $y_3''' = -e^{-t}$   $y_3^{IV} = e^{-t}$   $e^{-t} + 2(-e^{-t}) + e^{-t} = 0 \checkmark$

$y_4 = te^{-t}$   $y_4' = e^{-t} - te^{-t}$   $y_4'' = (-1)e^{-t} + (1-t)(-e^{-t})$   $y_4''' = e^{-t} - (t-2)e^{-t}$   $y_4^{IV} = -e^{-t} - (3-t)e^{-t}$

$$\begin{matrix} (1-t)e^{-t} \\ -(2-t)e^{-t} \\ (t-2)e^{-t} \end{matrix}$$

$$\begin{matrix} (3-t)e^{-t} \\ -e^{-t} - (3-t)e^{-t} \\ (t-4)e^{-t} \end{matrix}$$