

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Which elementary matrices are needed to transform the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ into $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$? Write each matrix and the order they are applied in.

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \quad -2R_1 + R_2 \rightarrow R_2$$

$$E_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} \quad \frac{1}{5}R_2 \rightarrow R_2$$

$$E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1/5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad R_2 + R_1 \rightarrow R_1$$

$$E_3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2. Use the information from problem #1, to find the LU factorization of A .

$$L = E_1^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix}$$

$$\text{or } L = E_1^{-1}E_2^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & 5 \end{bmatrix} \quad U = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

3. The matrix $P = \begin{bmatrix} .9 & .1 & .06 \\ .07 & .85 & .14 \\ .03 & .05 & .80 \end{bmatrix}$ represents the change of state matrix for cars picked up at rental locations on the Eastside, Southside and Westside of town respectively, and which location they are returned to. If the initial number of cars at each location on Sunday is given by $\vec{x} = \begin{bmatrix} 600 \\ 350 \\ 250 \end{bmatrix}$, how many cars will be at each location on Monday? How many will be at each location on Friday? (Round your final answers to whole numbers of cars.)

$$\text{Monday} = P\vec{x} = \begin{bmatrix} 590 \\ 375 \\ 235 \end{bmatrix} \quad \text{Friday} = P^5\vec{x} = \begin{bmatrix} 570 \\ 423 \\ 207 \end{bmatrix}$$