

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Find the linear combination of $\vec{v}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ needed to produce the vector

$$\vec{u} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|c} -1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & -1 & 2 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

\vec{u} cannot be written as a linear combination of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

2. Are $\vec{v}_1 = \begin{bmatrix} -4 \\ 4 \\ -1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -5 \\ 3 \\ 3 \end{bmatrix}$ linearly independent? Why or why not?

yes. 2 vectors can only be dependent if they are multiples of each other

3. Find an explicit description of $\text{Span}(A)$ —find the minimum spanning set—of $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$. Does the minimum spanning set form a basis? Why or why not?

$$\left[\begin{array}{cccc} 1 & -1 & 0 & 2 \\ 2 & 0 & 1 & 1 \\ 1 & 3 & 1 & 1 \end{array} \right] \Rightarrow \text{rref} \Rightarrow \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -5 \end{array} \right]$$

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ yes this is a basis. it spans \mathbb{R}^3 and is linearly independent.