MAT-222-840

Team Problems for Chapter 10

Name: Solutions

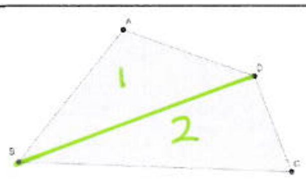
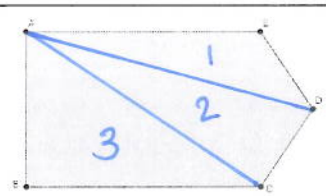
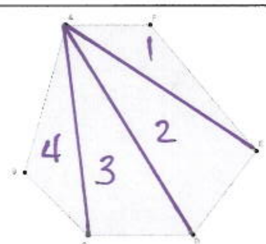
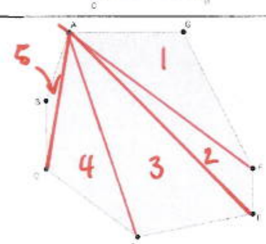
Date: ______________

Problem #1: Divide and Conquer

In this problem, we will sketch out a proof of the Polygon Interior-Angle Sum Theorem. That theorem says that the sum of the measures of the interior angles of a convex n-gon is: $\boxed{180(n-2)}$

We will start with the fact that the sum of the interior angles of a triangle is 180 degrees.

In each of the polygons below, draw diagonals from the vertex A to any vertices with which A does not share an edge. For example, in the quadrilateral, draw the diagonal $\overline{AC}$. This should divide the polynomial into a number of triangles. Count up the triangles and complete the table.

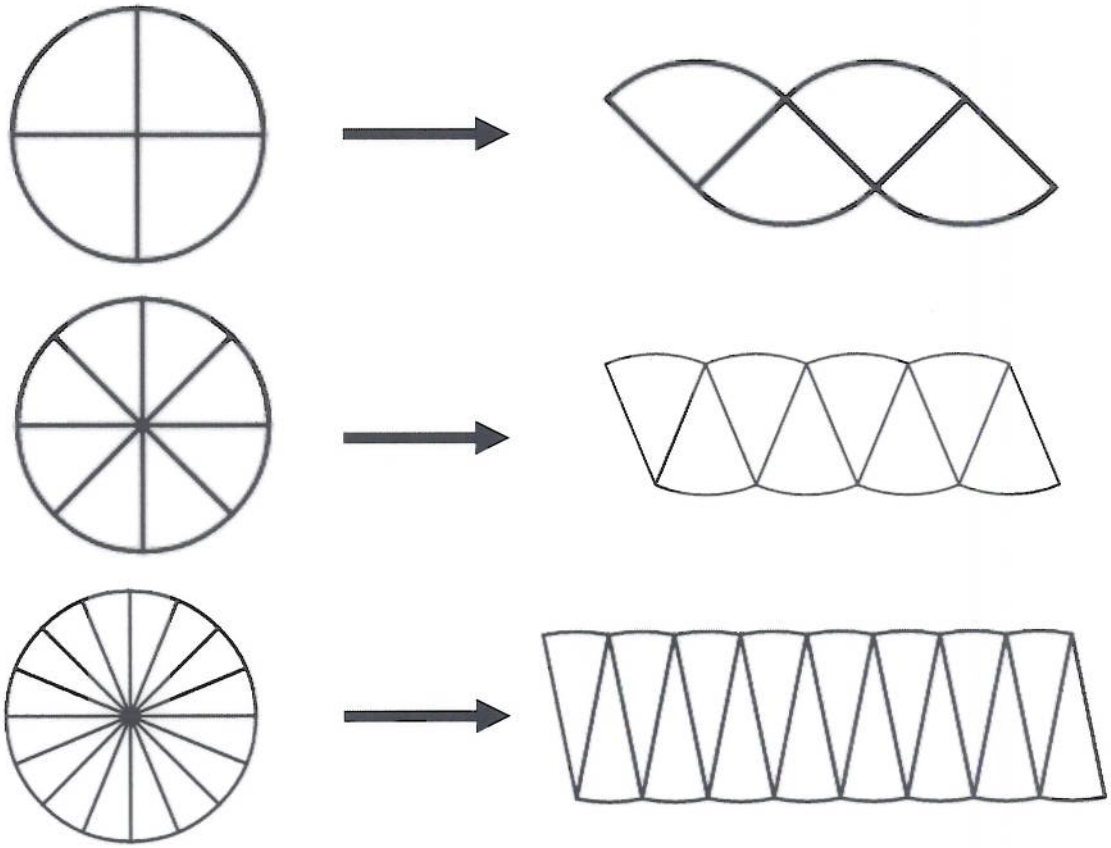
Polygon	Number of Sides	Number of triangles	**Total** number of degrees in triangles
	4	2	180(2)
	5	3	180(3)
	6	4	180(4)
	7	5	180(5)

In a polygon with n sides, drawing all diagonals from one vertex will divide the polygon into $\underline{n-2}$ triangles. Since each triangle has an interior angle sum of 180 degrees, the n-gon has an interior angle sum of

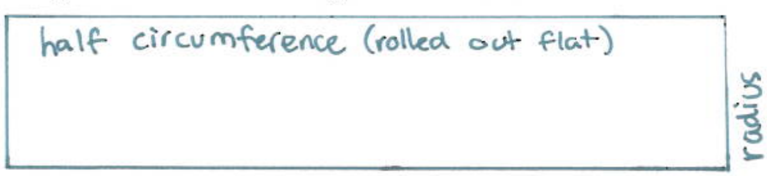
$\underline{\quad n-2 \quad}$ ×180 degrees
# of triangles

Problem #2: Why the Area Formula for Circles Makes Sense

1. Consider the pictures below, where the circle is cut into 4, 8, and 16 pie-pieces.



1. Visualize cutting the circle into more and more smaller pie pieces and rearranging them as above.
   - What shape would the rearranged circle become more and more like? Draw it below.



   - What would the lengths of the sides of this shape be? r, $\frac{1}{2}C = \pi r$
   - What would the area of this shape be? $A = bh = \pi r \cdot r = \pi r^2$

Using your answers to part 4, explain why it makes sense that a circle of radius r units has area πr^2 square units, given that the circumference of a circle of radius r is $2\pi r$.

Since we were just moving area around, the area of the original circle must be πr^2 as well.

Problem #3: Similar Areas Problem-Solving

1. To plan the renovation of an art gallery, a 1/10 scale model of the Pre-Revolution French Landscape Painting Wing was made. The warm ochre paint that the design company chose to paint the walls of the model cost $3.10. If the company uses the same paint, at the same cost, to paint the walls of the gallery, how much will it cost?

Paint → Area → scale is $\left(\frac{1}{10}\right)^2 = \frac{1}{100}$

$$\frac{1}{100} = \frac{3.10}{x}$$

$$\boxed{x = \$310}$$

2. Mrs. Henderson's neighbor just had his 1-acre property fenced in an attractive 6 foot picket for $7500. If Mrs. Henderson hires the same company to fence her 3-acre property with the same type of fence, how much will it cost her? Assume Mrs. Henderson's property and her neighbor's property are similar in shape.

$\frac{1}{3}$ compares acres (area)

To compare perimeter, use

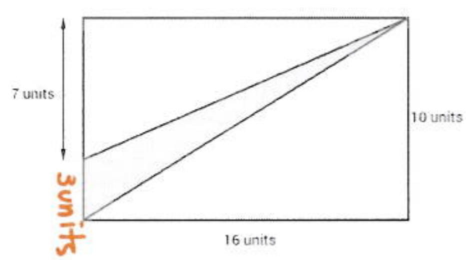
$\sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$

$$\frac{1}{\sqrt{3}} = \frac{\$7500}{x}$$

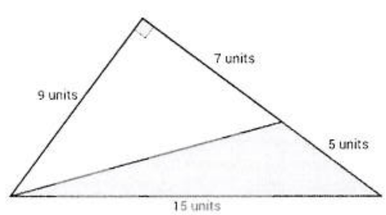
$$x = \$7500\sqrt{3} \approx \boxed{\$12990.38}$$

Problem #4: Determining Areas

Determine each of the shaded areas below.



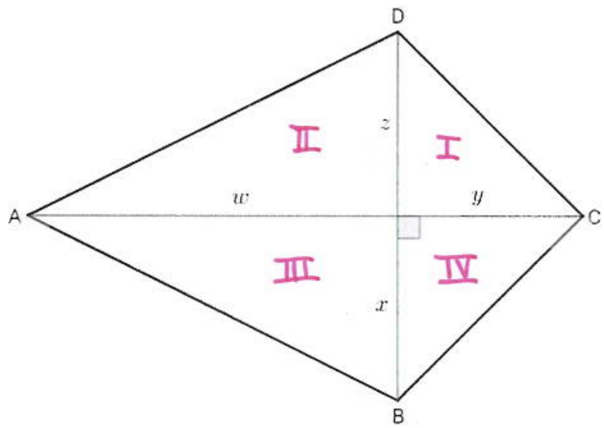
$$A = \frac{1}{2}bh = \frac{1}{2}(3)(16) = \boxed{24 \text{ units}^2}$$



$$A = \frac{1}{2}bh = \frac{1}{2}(5)(9) = \boxed{22.5 \text{ units}^2}$$

Problem #5: Why the Area formula for Kites makes sense

Consider the kite $ABCD$ below with diagonals d_1 and d_2 drawn in. The diagonals have been divided at their point of intersection into lengths such that $d_1 = w + y$ and $d_2 = x + z$.



1. Find the area of $ABCD$ by breaking it up into four triangles and adding those areas together.

I: $A = \frac{1}{2}yz$

II: $A = \frac{1}{2}wz$ Total Area $= \frac{1}{2}yz + \frac{1}{2}wz + \frac{1}{2}wx + \frac{1}{2}yx$

III: $A = \frac{1}{2}wx$

IV: $A = \frac{1}{2}yx$

2. Starting with the kite area formula $A = \frac{1}{2}d_1 d_2$, make the substitutions $d_1 = w + y$ and $d_2 = x + z$ and simplify by expanding (multiplying out).

$A = \frac{1}{2}(w+y)(x+z)$

$A = \frac{1}{2}(wx + wz + yx + yz)$

$A = \frac{1}{2}wx + \frac{1}{2}wz + \frac{1}{2}yx + \frac{1}{2}yz$

3. Verify that your area formulas in Step 1 and Step 2 are the same. ✓✓

4. Explain why this method would also verify the area formula for rhombuses is true.

The same area formula $A=\frac{1}{2}d_1d_2$ is used for rhombuses and diagonals likewise break a rhombus into 4 right triangles.