

**Problem #1: Relating the Kinds of Quadrilaterals**

In each of parts A. through D., describe the relationships among the given kinds of shapes. In each case, write as many sentences as you can of the following forms:

Every \_\_\_\_\_ is a \_\_\_\_\_.

There are \_\_\_\_\_ that are not \_\_\_\_\_.

- A. Describe the relationship between squares and rectangles.

Every square is a rectangle.

There are rectangles that are not squares.

- B. Describe the relationship between squares and rhombuses.

Every square is a rhombus.

There are rhombuses that are not squares.

- C. Describe the relationship between rectangles and trapezoids.

No rectangle is a trapezoid.

No trapezoid is a rectangle.

Some textbooks define trapezoids as "at least 1 pair of parallel sides", which would change these answers.

- D. Describe the relationship between rhombuses and parallelograms.

Every rhombus is a parallelogram.

There are parallelograms that are not rhombuses.

- E. Describe the relationship between parallelograms and rectangles.

Every rectangle is a parallelogram.

There are parallelograms that are not ~~rhomb~~ rectangles.

**Problem #2: Quadrilaterals with various attributes**

In each of the following A through E, you will answer the question "How many quadrilaterals have this particular attribute?"

- If you believe that there is exactly one such quadrilateral, draw it and explain why no other quadrilateral fits.
- If you believe that there is more than one such quadrilateral, draw a few examples and determine the number of quadrilaterals that have this attribute.
- If you believe that there are no such quadrilaterals, explain why.

How many quadrilaterals can you draw that have...

A. these four lengths: 2cm, 3cm, 4cm, 5cm?

- |               |               |
|---------------|---------------|
| 1. 2, 3, 4, 5 | 4. 2, 4, 5, 3 |
| 2. 2, 3, 5, 4 | 5. 2, 5, 3, 4 |
| 3. 2, 4, 3, 5 | 6. 2, 5, 4, 3 |

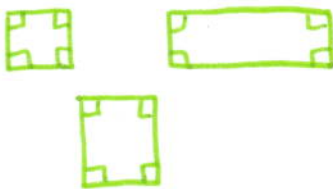
The pairs 1&6, 2&4, 3&5 are the same, in backwards order. Therefore there are 3 such quadrilaterals.

B. all sides equal?



These are rhombuses — infinitely many exist.

C. all angles equal?



These are rectangles — infinitely many exist.

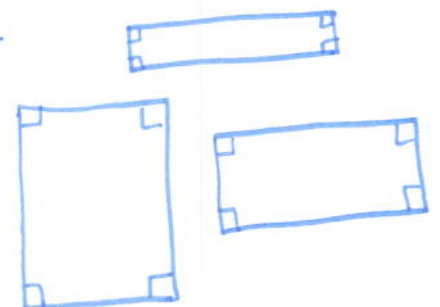
D. all sides equal but not all angles equal?

These are rhombuses that are not squares — infinitely many exist.



E. all angles equal but not all sides equal?

These are rectangles that are not squares — infinitely many exist.



**Problem #3: Quadrilaterals with various attributes (continued)**


In each of the following F. through K., you will answer the question "How many quadrilaterals have this particular attribute?"

- If you believe that there is exactly one such quadrilateral, draw it and explain why no other quadrilateral fits.
- If you believe that there is more than one such quadrilateral, draw a few examples and determine the number of quadrilaterals that have this attribute.
- If you believe that there are no such quadrilaterals, explain why.

How many quadrilaterals can you draw that have...


F. exactly two pairs of congruent sides, but each pair having different length?

Some parallelograms & ~~some~~<sup>all</sup> kites infinitely many




G. exactly 2 congruent sides?

Isosceles trapezoids & some others — infinitely many.



H. exactly 3 congruent sides?

Isosceles trapezoids with legs congruent to 1 base (infinitely many)



I. exactly 1 right angle?

Lots of kinds!



J. exactly 2 right angles?

Some kites & some trapezoids!



K. exactly 3 right angles?

None! Exactly 3 right angles means the 4<sup>th</sup> would be a right angle, too.

### Problem #4: Investigating Diagonals of Quadrilaterals

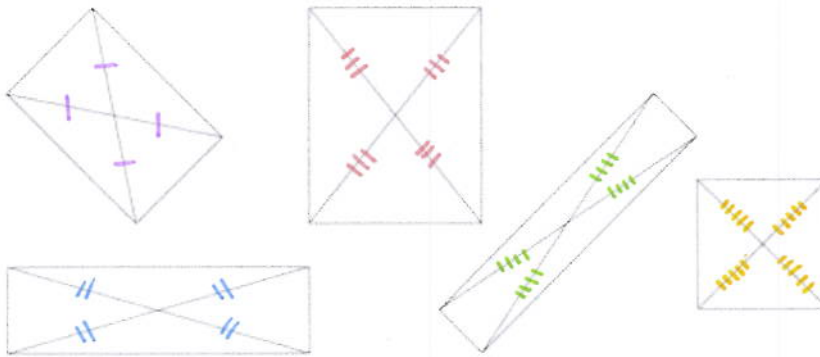
On this page there are many examples of different kinds of quadrilaterals and their diagonals. Look carefully at the diagonals and do the following:

- Observe and measure the angles that the diagonals make with each other.
- Observe where the diagonals meet: where is this point located on the diagonals?
- Compare the length of the two diagonals.

What do the diagonals of rectangles all have in common? What do the diagonals of rhombuses all have in common? What about diagonals of general quadrilaterals?

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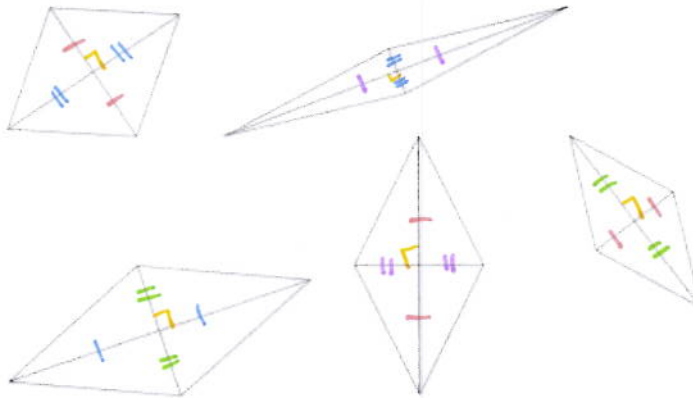
Rectangles:



- diagonals same length
- diagonals bisect each other

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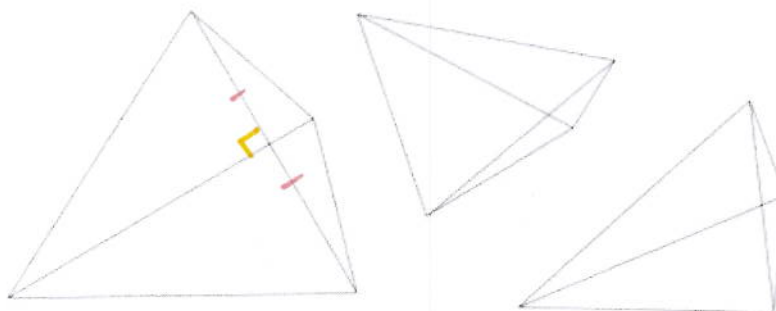
Rhombuses:



- diagonals bisect each other
- meet at right angles

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Other Quadrilaterals:

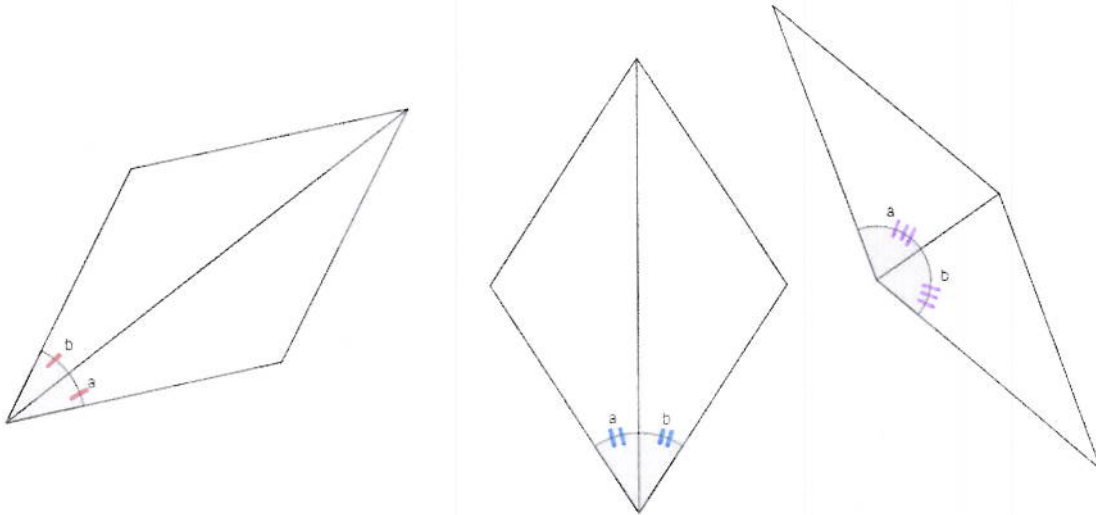


- Neither bisect each other nor do they necessarily meet at right angles.

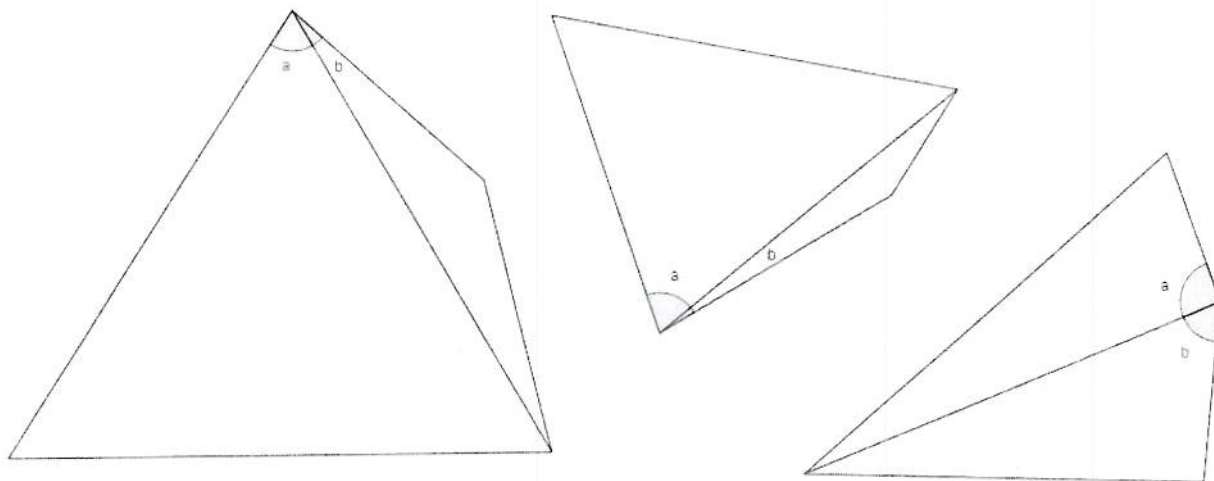
**Problem #5: More Investigating Diagonals**

Below there are some rhombuses and other quadrilaterals with one of the two diagonals drawn in. The two angles created by the quadrilaterals are labeled  $a$  and  $b$ . In each case, compare angle  $a$  with angle  $b$ . Compare what you observed for quadrilaterals with what you observed for rhombuses.

Rhombuses:



Other Quadrilaterals:



Diagonals of rhombuses bisect the rhombuses' angles.  
 Other quadrilaterals - not necessarily.