MATH 240, Exam #1, Summer 2017 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Let $\vec{u} = \langle 3, 1, 5\rangle$ and $\vec{v} = \langle -2, 3, 1\rangle$. Find (10 points)

$$\frac{2\vec{u} + 3\vec{v}}{\|\vec{u} + \vec{v}\|}$$

$2\vec{u} + 3\vec{v} = 2\langle 3,1,5\rangle + 3\langle -2,3,1\rangle$
$= \langle 6,2,10\rangle + \langle -6,9,3\rangle$
$= \langle 0, 11, 13\rangle$

$\vec{u}+\vec{v} = \langle 3,1,5\rangle + \langle -2,3,1\rangle$
$= \langle 1,4,6\rangle$

$\|\vec{u}+\vec{v}\| = \sqrt{1^2+4^2+6^2} = \sqrt{1+16+36}$
$= \sqrt{53}$

$$\frac{\langle 0, 11, 13\rangle}{\sqrt{53}}$$

2. Find the equation of the line that passes through the points $(3,4,1)$ and $(-5,2,7)$. Write your answer in: (8 points)

a. Parametric form

$\vec{r}(t) = (3-8t)\hat{i} + (4-2t)\hat{j} + (1+6t)\hat{k}$

$\vec{v} = \langle -5-3, 2-4, 7-1\rangle$
$= \langle -8, -2, 6\rangle$

b. Symmetric form

$$\frac{x-3}{-8} = \frac{y-4}{-2} = \frac{z-1}{6}$$

$x = 3-8t$
$y = 4-2t$
$z = 1+6t$

3. Find a set of parametric equations of the line through the point $(2,-3,6)$ that is parallel to the line $x=-2+3t, \quad y=8-9t, \quad z=7-t.$ (8 points)

$\vec{v} = \langle 3, -9, -1 \rangle$

$x = 2 + 3t$

$y = -3 - 9t$

$z = 6 - t$

4. Find the standard equation of the sphere that has $(5,3,-1)$ and $(3,7,9)$ as end points of the diameter. (8 points)

midpoint = center

$\left(\frac{5+3}{2}, \frac{3+7}{2}, \frac{-1+9}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}, \frac{8}{2}\right) = (4,5,4)$

distance $= \sqrt{(5-3)^2+(3-7)^2+(-1-9)^2}$ diameter

$= \sqrt{2^2+(-4)^2+(-10)^2} = \sqrt{4+16+100} = \sqrt{120} = 2\sqrt{30}$

radius $\frac{2\sqrt{30}}{2} = \sqrt{30}$

$(x-4)^2+(y-5)^2+(z-4)^2 = 30$

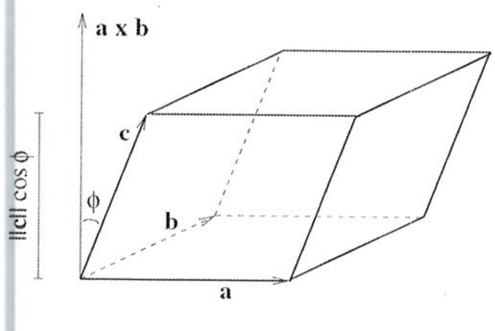
5. The projection of the vector $\vec{u}$ in the direction of $\vec{v}$ is given by $proj_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}$. Find the projection of the vector $\vec{u} = \langle 2,1,5\rangle$ in the direction of the vector $\vec{v} = \langle -9,3,2\rangle$. (8 points)

$\vec{u}\cdot\vec{v} = -18+3+10 = -5$

$\vec{v}\cdot\vec{v} = 81+9+4 = 94$

$proj_{\vec{v}}\vec{u} = \frac{-5}{94}\langle -9,3,2\rangle$

6. Consider the parallelepiped (slanted box) determined by the position vectors $\vec{a} = \langle -2,5,1\rangle, \vec{b} = \langle 1,3,-1\rangle, \vec{c} = \langle 1,4,6\rangle$. The volume is given by the triple scalar product $|\vec{c}\cdot(\vec{a}\times\vec{b})| = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$. Find the volume of the parallelepiped. (10 points)



$\begin{vmatrix} -2 & 5 & 1 \\ 1 & 3 & -1 \\ 1 & 4 & 6 \end{vmatrix} =$

$-2\begin{vmatrix} 3 & -1 \\ 4 & 6 \end{vmatrix} - 5\begin{vmatrix} 1 & -1 \\ 1 & 6 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} =$

$-2(18+4) - 5(6+1) + 1(4-3) =$

$-2(22) - 5(7) + 1(1) =$

$-44 - 35 + 1 = -78$

$\boxed{78}$

7. Three people located at A, B, C pull on ropes tied to a ring. If A and B are pulling as shown, find the magnitude and direction with which C must pull so that no one moves (system is in equilibrium). You may round your final answer to 2 decimal places. (12 points)

$F_A = \langle 20\cos 15°, 20 \sin 15° \rangle$

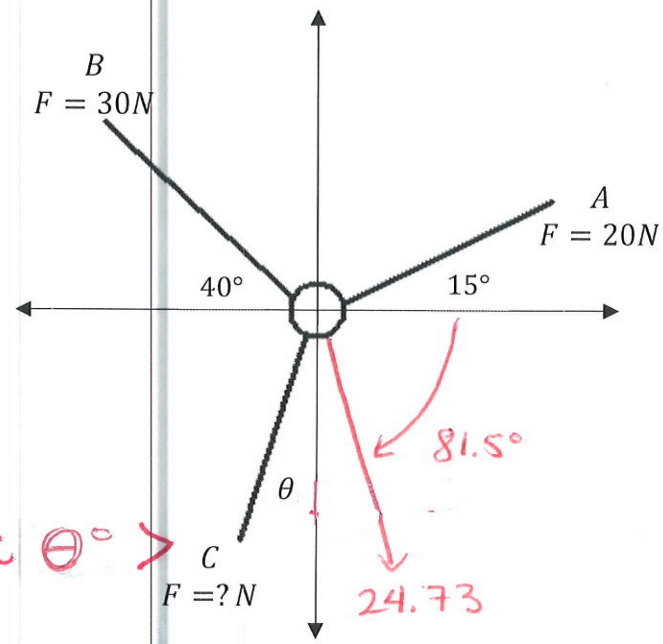
$F_B = \langle 30\cos 140°, 30 \sin 140° \rangle$

$F_A + F_B = \langle -3.66, 24.46 \rangle$

$F_C = \langle 3.66, -24.46 \rangle$

$\|F_C\| = 24.73$

$F_C = \langle 24.73 \cos \theta°, 24.73 \sin \theta° \rangle$



$\theta = \tan^{-1}\left(\frac{-24.46}{3.66}\right) = -81.5° = 278.5°$

or $\theta = -\cos^{-1}\left(\frac{3.66}{24.73}\right)$

8. Consider a particle traveling along a path determined by $\vec{r}(t) = 5t\hat{\imath} + 3\sin t\,\hat{\jmath} + 4\cos t\,\hat{k}$.

a. Find the velocity of the object. (3 points)

$\vec{r}'(t) = 5\hat{\imath} + 3\cos t\,\hat{\jmath} - 4\sin t\,\hat{k}$

b. What is the speed of the object? (3 points)

$\|\vec{r}'(t)\| = \sqrt{25 + 9\cos^2 t + 16\sin^2 t} = \sqrt{34 + 7\sin^2 t}$

c. What is the acceleration of the object? (3 points)

$\vec{r}''(t) = 0\hat{i} - 3\sin t\,\hat{j} - 4\cos t\,\hat{k}$

9. A volleyball is hit when it is 4 feet off the ground and 20 feet from a 6-foot-high net. It leaves the point of impact with an initial velocity of 24 ft/sec at an angle of 45° and slips by the opposing team untouched.

a. Find a vector equation for the path of the volleyball. (4 points)

$\vec{r}(t) = (v_0 t \cos\theta)\hat{i} + (-\frac{1}{2}gt^2 + v_0 t \sin\theta + h_0)\hat{j}$

$(24\cos 45°)t\hat{i} + (-16t^2 + 24\sin 45° \cdot t + 4)\hat{j}$

$12\sqrt{2}t\,\hat{i} + (-16t^2 + 12\sqrt{2}t + 4)\hat{j}$

b. How high does the volleyball go? (4 points)

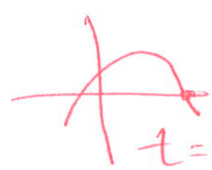
$-32t + 12\sqrt{2} = 0$

$\frac{12\sqrt{2}}{32} = t \approx .53033$

$y = 8.5$ ft

c. Find its range. (4 points)

$1.26\,(12\sqrt{2}) = 21.4$ feet



$t = 1.26$

d. When is the volleyball 8 feet above the ground? (4 points)

$t \approx .354$ seconds

$t \approx .7071$ Seconds