

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Let $\vec{u} = \langle 1, 2, 4 \rangle$ and $\vec{v} = \langle -1, 0, 8 \rangle$. Find

(10 points)

$$\begin{aligned}\vec{u} - 4\vec{v} &= \langle 1, 2, 4 \rangle - 4\langle -1, 0, 8 \rangle \\ &= \langle 1, 2, 4 \rangle + \langle 4, 0, -32 \rangle \\ &= \langle 5, 2, -28 \rangle\end{aligned}$$

$$\frac{\vec{u} - 4\vec{v}}{\|\vec{u} - \vec{v}\|}$$

$$= \frac{\langle 5, 2, -28 \rangle}{2\sqrt{6}}$$

$$\begin{aligned}\vec{u} - \vec{v} &= \langle 1, 2, 4 \rangle - \langle -1, 0, 8 \rangle \\ &= \langle 2, 2, -4 \rangle\end{aligned}$$

$$\begin{aligned}\|\vec{u} - \vec{v}\| &= \sqrt{2^2 + 2^2 + (-4)^2} \\ &= \sqrt{4 + 4 + 16} = \sqrt{24} \\ &= 2\sqrt{6}\end{aligned}$$

2. Find the equation of the line that passes through the points $(2, 2, 6)$ and $(1, -3, 4)$. Write your answer in:

(8 points)

- a. Parametric form

$$\vec{r}(t) = (2-t)\hat{i} + (2-5t)\hat{j} + (6-2t)\hat{k}$$

- b. Symmetric form

$$\frac{x-2}{-1} = \frac{y-2}{-5} = \frac{z-6}{-2}$$

$$\begin{aligned}\vec{v} &= \langle 1-2, -3-2, 4-6 \rangle = \\ &= \langle -1, -5, -2 \rangle\end{aligned}$$

$$\begin{aligned}x &= 2-t \\ y &= 2-5t \\ z &= 6-2t\end{aligned}$$

3. Find a set of parametric equations of the line through the point $(1, 0, -11)$ that is parallel to the line $x = 5 + t$, $y = -5 + 6t$, $z = 3$. (8 points)

$$\vec{v} = \langle 1, 6, 0 \rangle$$

$$x = 1 + t, y = 6t, z = -11$$

4. Find the standard equation of the sphere that has $(3, 1, -3)$ and $(1, 5, 7)$ as end points of the diameter. (8 points)

midpoint = center

$$\left(\frac{3+1}{2}, \frac{1+5}{2}, \frac{-3+7}{2} \right) = \left(\frac{4}{2}, \frac{6}{2}, \frac{4}{2} \right) = (2, 3, 2)$$

$$\begin{aligned} \text{distance} &= \sqrt{(3-1)^2 + (1-5)^2 + (-3-7)^2} = \text{diameter} \\ &= \sqrt{2^2 + (-4)^2 + (-10)^2} = \sqrt{4 + 16 + 100} = \frac{\sqrt{120}}{2} \\ &= \frac{2\sqrt{30}}{2} = \sqrt{30} \end{aligned}$$

$$(x-2)^2 + (y-3)^2 + (z-2)^2 = 30$$

5. The projection of the vector \vec{u} in the direction of \vec{v} is given by $\text{proj}_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}$. Find the projection of the vector $\vec{u} = \langle 3, -2, 9 \rangle$ in the direction of the vector $\vec{v} = \langle 7, -3, 4 \rangle$. (8 points)

$$\vec{u}\cdot\vec{v} = 21 + 6 + 36 = 63$$

$$\vec{v}\cdot\vec{v} = 49 + 9 + 16 = 74$$

$$\text{proj}_{\vec{v}}\vec{u} = \frac{63}{74} \langle 7, -3, 4 \rangle$$

6. Consider the parallelepiped (slanted box) determined by the position vectors $\vec{a} = \langle 3, -5, 6 \rangle$, $\vec{b} = \langle 7, 1, 1 \rangle$, $\vec{c} = \langle 4, 3, 1 \rangle$. The volume is given by the triple scalar product $|\vec{c} \cdot (\vec{a} \times \vec{b})| =$

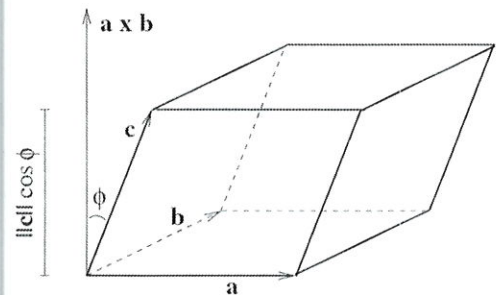
$$\begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}. \text{ Find the volume of the parallelepiped. (10 points)}$$

$$\begin{vmatrix} 3 & -5 & 6 \\ 7 & 1 & 1 \\ 4 & 3 & 1 \end{vmatrix} =$$

$$3 \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 5 \begin{vmatrix} 7 & 1 \\ 4 & 1 \end{vmatrix} + 6 \begin{vmatrix} 7 & 1 \\ 4 & 3 \end{vmatrix} =$$

$$3(1-3) + 5(7-4) + 6(21-4) =$$

$$3(-2) + 5(3) + 6(17) = -6 + 15 + 102 = 111$$



7. Three people located at A, B, C pull on ropes tied to a ring. If A and B are pulling as shown, find the magnitude and direction with which C must pull so that no one moves (system is in equilibrium). You may round your final answer to 2 decimal places. (12 points)

$$F_A = \langle 10 \cos 20^\circ, 10 \sin 20^\circ \rangle$$

$$F_B = \langle 40 \cos 135^\circ, 40 \sin 135^\circ \rangle$$

$$F_C = ? \quad F_A + F_B + F_C = 0$$

$$F_A + F_B = \langle -18.887, 31.70447 \rangle$$

$$\|F_A + F_B\| \approx 36.9038$$

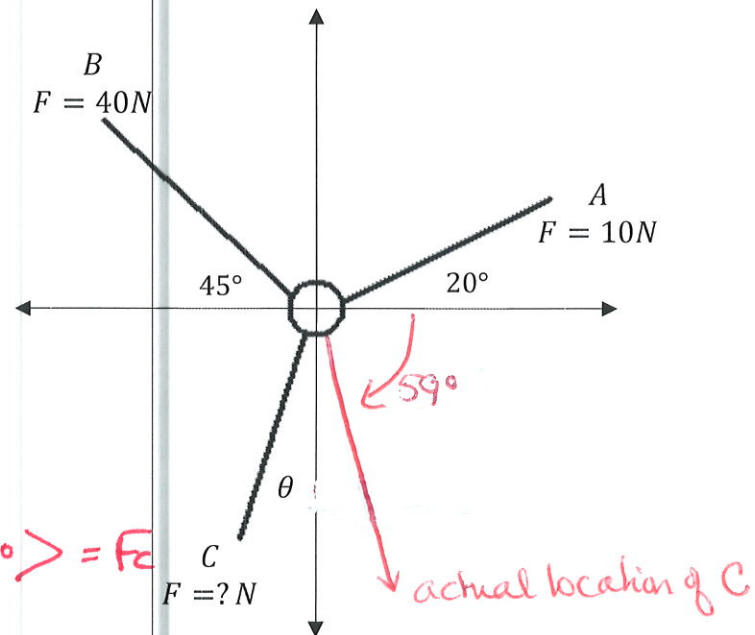
$$\langle 36.9038 \cos \theta^\circ, 36.9038 \sin \theta^\circ \rangle = F_C$$

$$\vec{F}_C = \langle 18.887, -31.704 \rangle \leftarrow \text{this vector is in QIV}$$

$$\langle 18.89, -31.70 \rangle$$

$$\theta = \tan^{-1} \left(\frac{-31.7044}{18.887} \right) = -59^\circ$$

$$F_C = 36.9 \text{ N at } 300.8^\circ \text{ (roughly } 301^\circ)$$



8. Consider a particle traveling along a path determined by $\vec{r}(t) = \sin t \hat{i} + 2 \cos t \hat{j} + 2t \hat{k}$.
- a. Find the velocity of the object. (3 points)

$$\vec{v}'(t) = \cos t \hat{i} - 2 \sin t \hat{j} + 2 \hat{k}$$

- b. What is the speed of the object? (3 points)

$$\begin{aligned} \|\vec{v}'(t)\| &= \sqrt{\cos^2 t + 4 \sin^2 t + 4} \\ &= \sqrt{5 + 3 \sin^2 t} \end{aligned}$$

c. What is the acceleration of the object? (3 points)

$$\vec{r}''(t) = -g \sin t \hat{i} - 2 \cos t \hat{j} + 0 \hat{k}$$

9. A volleyball is hit when it is 3 feet off the ground and 10 feet from a 6-foot-high net. It leaves the point of impact with an initial velocity of 40 ft/sec at an angle of 60° and slips by the opposing team untouched.

a. Find a vector equation for the path of the volleyball. (4 points)

$$\begin{aligned} \vec{r}(t) &= (v_0 t \cos \theta) \hat{i} + \left(-\frac{1}{2} g t^2 + v_0 t \sin \theta + h_0\right) \hat{j} \\ &= (40 \cos 60^\circ) t \hat{i} + (-16 t^2 + 40 \sin 60^\circ \cdot t + 3) \hat{j} \\ &= 20 t \hat{i} + (-16 t^2 + 20\sqrt{3} t + 3) \hat{j} \end{aligned}$$

b. How high does the volleyball go? (4 points)

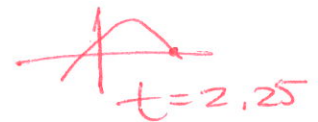
$$-32t + 20\sqrt{3} = 0$$

$$\frac{32t}{32} = \frac{20\sqrt{3}}{32} \Rightarrow t \approx 1.08$$

max y is ≈ 21.75 ft. (plug into \hat{j} coord)

c. Find its range. (4 points)

$$20(2.25) \approx 45 \text{ ft.}$$



plug into \hat{i} coord.

d. When is the volleyball 10 feet above the ground? (4 points)

$$\begin{aligned} t &= .226 \text{ seconds} \\ t &= 1.939 \text{ seconds} \end{aligned}$$

