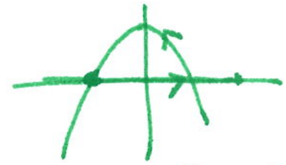
Math 240, Quiz #12, Summer 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Evaluate the line integral $\int_C 2xy\,dx + (x+y)\,dy$ boundary of region between $y=0, y=1-x^2$ two ways.

   a. By parametrizing the curve and integrating along the curve.



$$\int_0^2 2(-1+t)(0)(1)\,dt + [(-1+t)+0](0)\,dt = 0$$

$$\int_1^{-1} 2(t)(1-t^2)(1)\,dt + [(t)+(1-t^2)](-2t)\,dt \qquad \vec{r}_1(t) = (-1+t)\hat{\imath} \quad [0,2]$$

$$\vec{r}_2(t) = t\,\hat{\imath} + (1-t^2)\hat{\jmath} \quad [1,-1]$$

$$\int_1^{-1} \cancel{2t} - \cancel{2t^3}\,dt + (-2t^2 - \cancel{2t} + \cancel{2t^3})\,dt =$$

$$\int_1^{-1} -2t^2\,dt = \int_{-1}^{1} 2t^2\,dt = \int_{-1}^{1} 2t^2\,dt$$

$$= 2\int_0^1 2t^2\,dt = 2\left[\tfrac{2}{3}t^3\right|_0^1 = \frac{4}{3}$$

   b. By using Green's Theorem.

$$\iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dy\,dx$$

$$\int_{-1}^{1}\int_0^{1-x^2} 1 - 2x\,dy\,dx =$$

$$\int_{-1}^{1} y - 2xy \Big|_0^{1-x^2} dx = \int_{-1}^{1} 1-x^2 - 2x(1-x^2)\,dx$$

$$-2x + 2x^3$$

$$= \int_{-1}^{1} 1 - 2x - x^2 + 2x^3\,dx = \int_{-1}^{1} 1 - x^2\,dx + \int_{-1}^{1} \overset{0 \text{ odd}}{\cancel{-2x + 2x^3}}\,dx$$

$$2\int_0^1 1 - x^2\,dx = 2\left[x - \tfrac{1}{3}x^3\right]_0^1 = 2\left[1 - \tfrac{1}{3}\right] = \frac{4}{3}$$